Semantics Bootcamp Part III: More of English

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1 Fun with Functional Application

Composition rules

For branching nodes:

(1) **Functional Application (FA)**
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any assignment a, if ⟦β⟧ is a function whose domain contains ⟦γ⟧, then ⟦α⟧ = ⟦β⟧(⟦γ⟧).

For non-branching nodes:

(2) **Non-branching Nodes (NN)**
If α is a non-branching node and β its daughter, then, for any assignment a, ⟦α⟧=⟦β⟧.

(3) **Terminal Nodes (TN)**
If α is a terminal node occupied by a lexical item, then ⟦α⟧ is specified in the lexicon.

1.1 Rick Perry is conservative

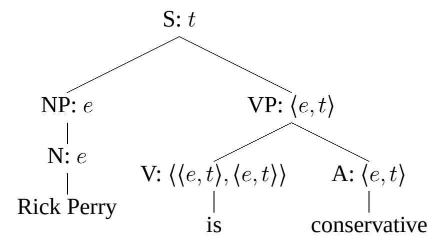
(4) ⟦Rick Perry⟧ = Rick Perry

(5) ⟦conservative⟧ = λx ∈ D_e . x is conservative

What to do with *is*? How about just an identity function:

(6) ⟦is⟧ = λf ∈ $D_{\langle e,t\rangle}$. f

(7)



⟦[$_S$ [$_{NP}$ [$_N$ Rick Perry]] [$_{VP}$ [$_V$ is] [$_A$ conservative]]]⟧

= ⟦ [$_{VP}$ [$_V$ is] [$_A$ conservative]]⟧(⟦ [$_{NP}$ [$_N$ Rick Perry]]⟧) FA

= ⟦[$_V$ is]⟧(⟦[$_A$ conservative]⟧)(⟦ [$_{NP}$ [$_N$ Rick Perry]]⟧) FA

= ⟦is⟧(⟦conservative⟧)(⟦Rick Perry⟧) NN

= [λf ∈ $D_{\langle e,t\rangle}$. f](λx ∈ D_e . x is conservative)](Rick Perry) TN

= λx ∈ D_e . x is conservative(Rick Perry) β-reduction

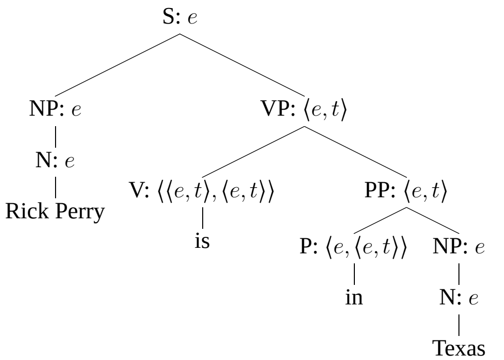
= 1 iff Rick Perry is conservative* (β-reduction)

*Notice that I wrote "= 1 iff Rick Perry is conservative" instead of "= Rick Perry is conservative". Normally, the result of β-reduction is written by replacing the λ-bound variable by the argument to which the function is being applied, but this case is different. That is because for Heim & Kratzer, "$[\lambda\alpha : \phi \,.\, \gamma]$" is read as either (i) or (ii), "whichever makes sense":
(i) "the function which maps every α such that ϕ to γ"
(ii) "the function which maps every α such that ϕ to 1, if γ, and to 0 otherwise"
When the value description is a sentence, it makes sense to read it in way (ii).

1.2 Rick Perry is in Texas

(8) ⟦in⟧ = λy ∈ D_e . λx ∈ D_e . x is in y

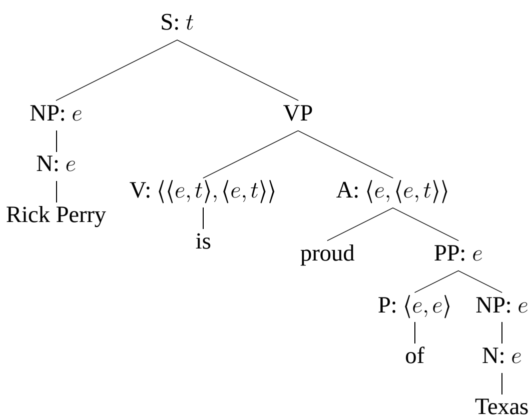
(9) ⟦Texas⟧ = Texas



1.3 Rick Perry is proud of Texas

(10) ⟦proud⟧ = λy ∊ D_e . λx ∊ D_e . x is proud of y

(11) ⟦of⟧ = λx ∊ D_e . x



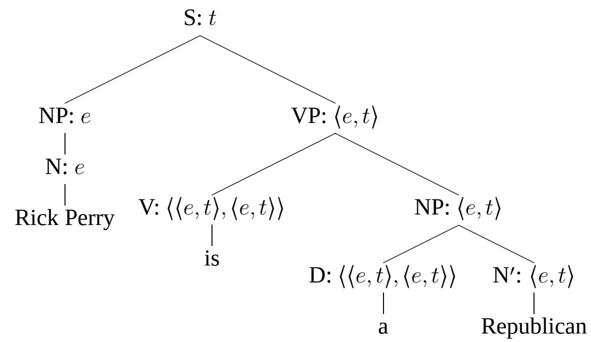
1.4 Rick Perry is a Republican

(12) ⟦Republican⟧ = λx ∊ D_e . x is a Republican

Hey, let's make the indefinite article vacuous too:

(13) ⟦a⟧ = λf ∊ $D_{\langle e,t\rangle}$. f

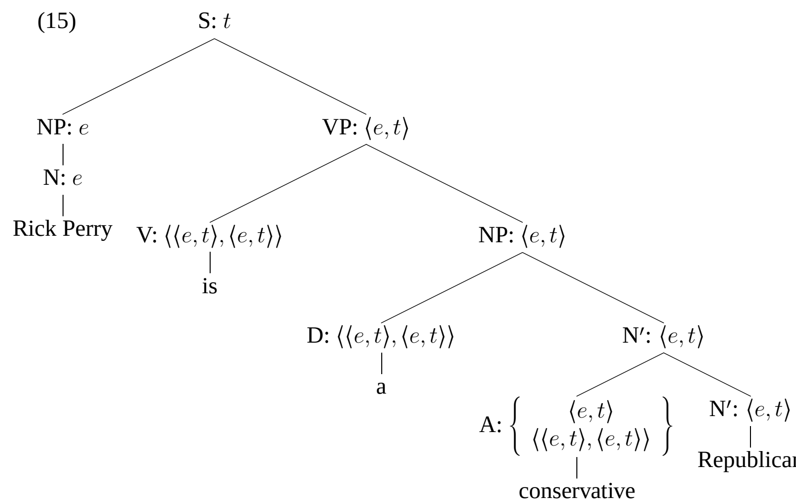
(14)



2 Predicate Modification

2.1 Rick Perry is a conservative Republican

(15)



Our lexical entry for *conservative* from above is type $\langle e,t \rangle$:

(16) $[\![\text{conservative}_1]\!] = \lambda x \in D_e$. x is conservative

If we want to use Functional Application here, we need *conservative* to be a function of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$.

(17) $[\![\text{conservative}_2]\!] = \lambda f \in D_{\langle e,t\rangle}$. $\lambda x \in D_e$. f(x) = 1 and x is conservative

Now, $[\![[_{N'}\ [_A \text{ conservative}_2\ ]\ [_N \text{ Republican }]\ ]]\!]$

$= [\![[_A \text{ conservative}_2\ ]]\!]([\![[_N \text{ Republican }]]\!])$ FA

$= [\![\text{conservative}_2]\!]([\![\text{Republican}]\!])$ NN

$= \boxed{\lambda f \in D_{\langle e,t\rangle} . \lambda x \in D_e . f(x) = 1 \text{ and x is conservative}}\left(\boxed{\lambda x \in D_e . \text{ x is a Republican}}\right)$ TN

$= \lambda x \in D_e$. x is a Republican and x is conservative β-reduction

This is Montague's strategy. All adjectives are $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ for him, and in predicate position, they combine with a silent noun. Despite the ungrammaticality of **Rick Perry is conservative Republican*.

Alternative strategy: Use another composition rule.

(18) **Predicate Modification** (PM)
If α is a branching node, $\{\beta,\gamma\}$ is the set of α's daughters, and $[\![\beta]\!]$ and $[\![\gamma]\!]$ are both in $D_{\langle e,t\rangle}$, then $[\![\alpha]\!] = \lambda x \in D_e . [\![\beta]\!](x) = [\![\gamma]\!](x) = 1$

Now, $[\![[_{N'}\ [_A \text{ conservative}_1\ ]\ [_N \text{ Republican }]\ ]]\!]$

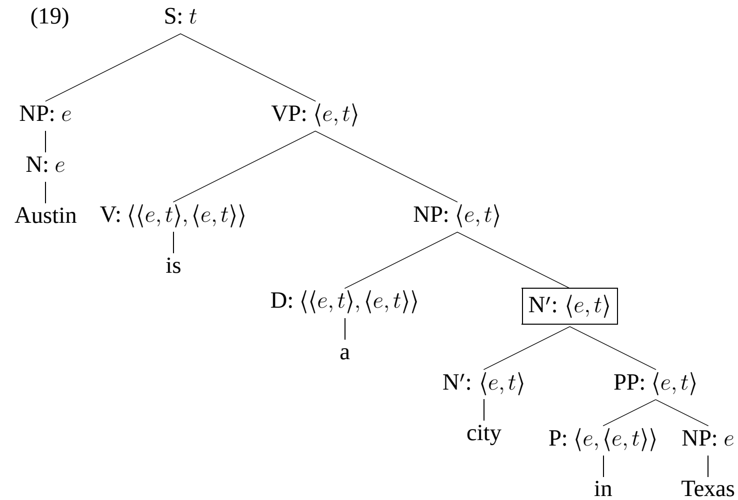
$= \lambda x \in D_e$. $[\![[_A \text{ conservative}_1\ ]]\!](x) = [\![[_N \text{ Republican }]]\!](x) = 1$ PM

$= \lambda x \in D_e$. $[\![\text{conservative}_1]\!](x) = [\![\text{Republican}]\!](x) = 1$ NN

$= \lambda x \in D_e$. x is conservative and x is a Republican β-reduction

2.2 Austin is a city in Texas

We can also use Predicate Modification with PP modifiers:

(19)


3 The definite article

What if we have *the governor of Texas* instead of *Rick Perry*?

3.1 The negative square root of 4

Regarding *the negative square root of 4*, Frege says, "We have here a case in which out of a concept-expression [i.e., an expression whose meaning is of type $\langle e,t\rangle$] a compound proper name is formed [that is to say, the entire expression is of type e] with the help of the definite article in the singular, which is at any rate permissible when one and only one object falls under the concept."

"Permissible": *the* denotes a function of type $\langle\langle e,t\rangle,e\rangle$ that is only *defined* for input predicates that characterize one single entity. In other words, *the* presupposes existence and uniqueness. Heim and Kratzer implement this as a restriction on the domain using the colon/dot notation:

(20) $[\![\text{the}]\!] = \lambda f \in D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1

So *the* is not a function from $D_{\langle e,t\rangle}$ to D_e; it is a *partial function* from $D_{\langle e,t\rangle}$ to D_e. But we can still give it the type $\langle\langle e,t\rangle, e\rangle$ if we interpret this to allow partial functions.

To flesh out Frege's analysis of this example further, Heim and Kratzer suggest that *square root* is a "transitive noun", with a meaning of type $\langle e, \langle e,t\rangle\rangle$, and that "*of* is vacuous, [[**square root**]] applies to 4 via Functional Application, and the result of that composes with [[**negative**]] *under predicate modification.*"

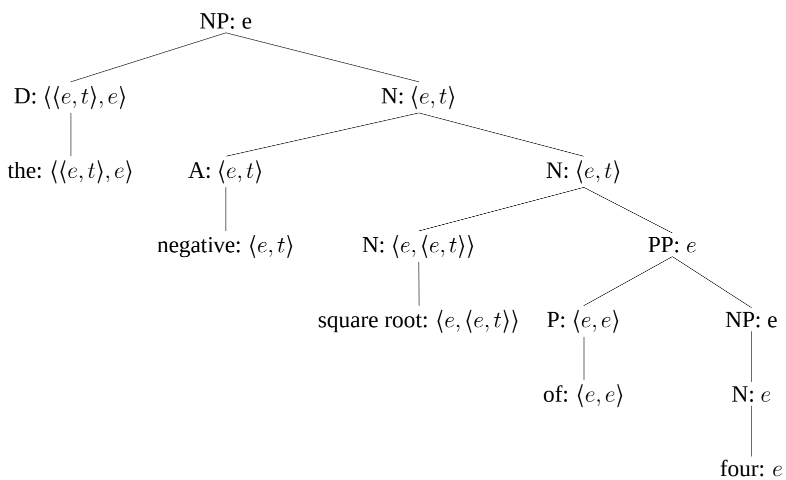
(21) ⟦negative⟧ = λx ∊ D_e . x is negative

(22) ⟦square root⟧ = λy ∊ D_e . λx ∊ D_e. x is the square root of y

(23) ⟦of⟧ = λx ∊ D_e . x

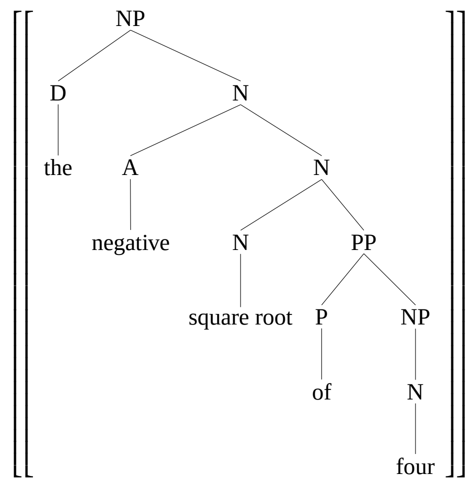
(24) ⟦four⟧ = 4

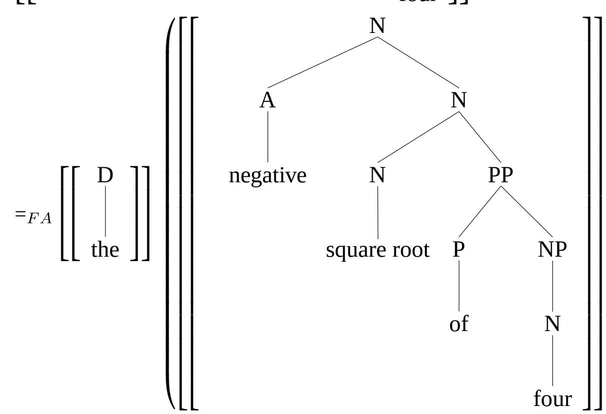
So the constituents will have denotations of the following types:

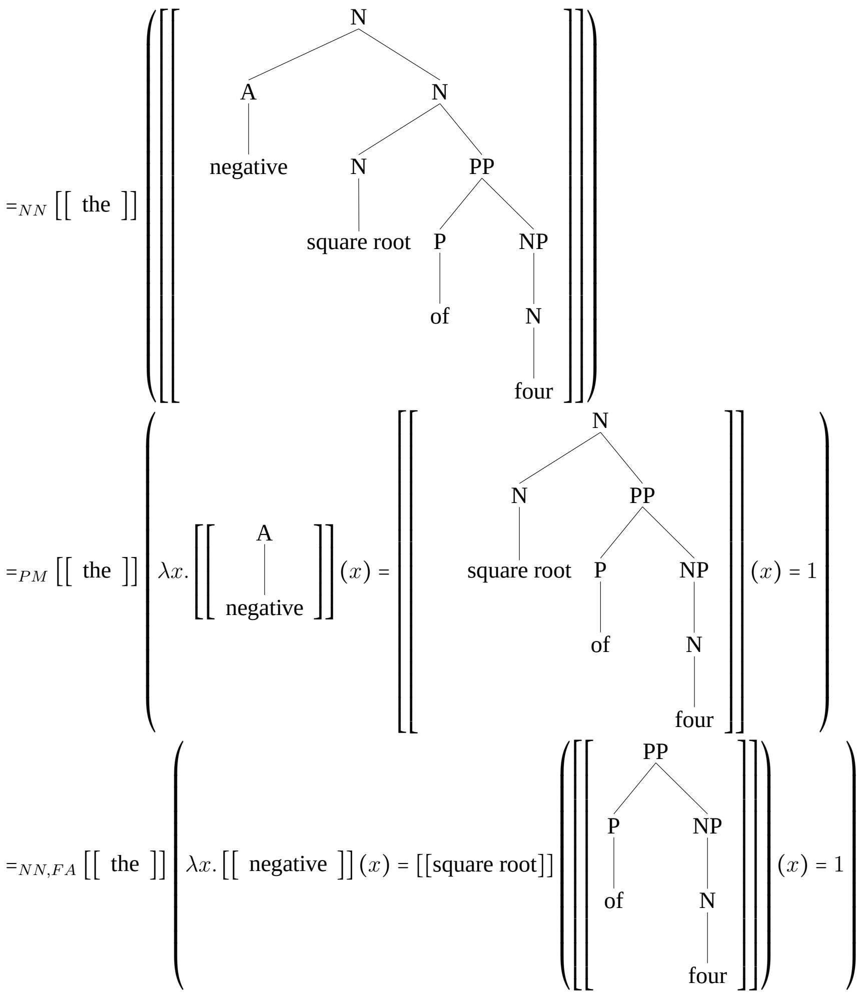


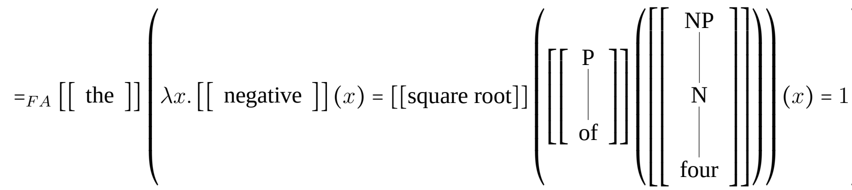
3.2 Top-down evaluation

To compute the value "top-down", we put the whole tree in one big old pair of denotation brackets, and use composition rules to break down the tree. Here I am putting the name of the rule I used as a subscript on the equals sign, because I don't have enough room to put them off to the right.









$=_{NN}$ [[the]] $(\lambda x.$[[negative]](x) = [[square root]]([[of]]([[four]]))$(x) = 1)$

Now we are done breaking down the tree. No more composition rules.

⟦the⟧(λx . [[negative]](x) = [[square root]]([[of]]([[four]]))(x) = 1)

= ⟦the⟧(λx . [[negative]](x) = [[square root]]([[of]](4))(x) = 1)

= ⟦the⟧(λx . [[negative]](x) = [[square root]]([λx . x](4))(x) = 1)

= ⟦the⟧(λx . [[negative]](x) = [[square root]](4)(x) = 1)

= ⟦the⟧(λx . [[negative]](x) = [λy . λz . z is a square root of y](4)(x) = 1)

= ⟦the⟧(λx . [[negative]](x) = [λz . z is a square root of 4](x) = 1)

= ⟦the⟧(λx . [[negative]](x) = 1 and x is a square root of 4)

= ⟦the⟧(λx . [[λz. z is negative]](x) = 1 and x is a square root of 4)

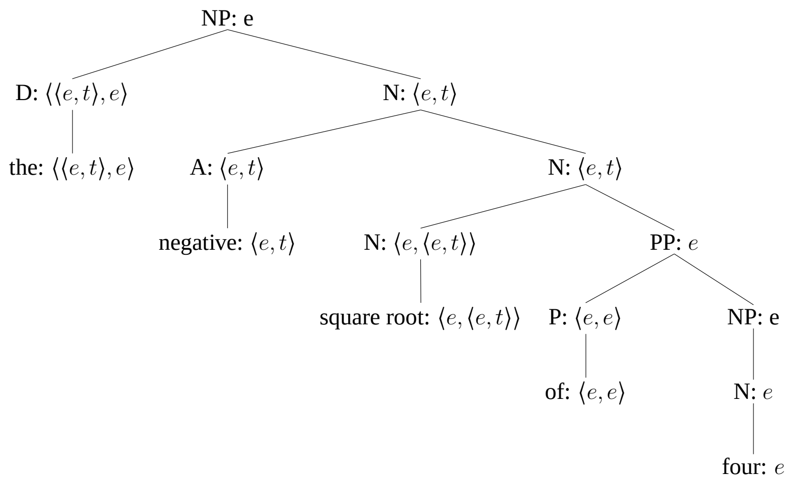
= ⟦the⟧(λx . x is negative and x is a square root of 4)

= [λf ∈ $D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1](λx . x is negative and x is a square root of 4)

= the unique y such that y is negative and x is a square root of 4

= -2

3.3 Bottom-up style



We need to compute a semantic value for every subtree/node. For convenience, we can group the nodes according to the string of words that they dominate, and start from the most deeply embedded part of the tree.

Note: I reserve the right to abbreviate e.g. $[_{PP}\ [_{P}$ of] $[_{NP}\ [_{N}$ four]]] as $[_{PP}$ of four], omitting all but the outermost brackets.

Bottom-up derivation

four

- ⟦four⟧ = 4 — TN
- ⟦$[_{NP}\ [_{N}$ four]]⟧ = ⟦$[_{N}$ four]⟧ = ⟦four⟧ = 4 — NN

of

- ⟦$[_{P}$ of]⟧ = ⟦of⟧ = λx . x — TN, NN

of four

- ⟦$[_{PP}$ of four]⟧ = ⟦$[_{P}$ of]⟧(⟦$[_{NP}$ 4]⟧) — FA
  = [λx . x](4) = 4

square root

- ⟦$[_{N}$ square root]⟧ = ⟦square root⟧ — NN
  = λy ∈ D_e . λx ∈ D_e. x is a square root of y — TN

square root of four

- ⟦$[_{N}$ square root of four⟧ = ⟦$[_{N}$ square root]⟧(⟦$[_{PP}$ of four]⟧) — FA
  [= λy ∈ D_e . λx ∈ D_e. x is a square root of y](4)
  = λx ∈ D_e. x is a square root of 4
  = λx ∈ D_e. x ∈ {2,-2}

negative

- ⟦$[_{A}$ negative]⟧ = ⟦negative⟧ = λx ∈ D_e . x is negative — TN, NN

negative square root of four

- ⟦$[_{N}$ negative square root of four]⟧
  = λx . ⟦$[_{A}$ negative]⟧(x) = ⟦$[_{N}$ square root of four]⟧(x)=1 — PM
  = λx. x is a square root of 4 and x is negative
  = λx. x ∈ {-2}

the

- ⟦$[_{D}$ the]⟧ = ⟦the⟧ — TN, NN
  = λf ∈ $D_{\langle e,t\rangle}$: there is exactly one x s. t. f(x) = 1 . the y such that f(y) = 1

the negative square root of four

- ⟦$[_{NP}$ the negative square root of four]⟧
  = ⟦$[_{D}$ the]⟧(⟦$[_{N}$ negative square root of four]⟧) — FA
  = [λf ∈ $D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1](λx. x ∈ {-2})
  = the unique y such that y ∈ {-2}
  = -2

Note that we used the same composition rules as we did using the top-down style!

This style is a little bit simpler and cleaner. But it doesn't work when you have to manipulate variable assignments... which we will cover tomorrow!