It is also interesting to note that there is a close connection between the treatment of disjunction and existential quantification in InqB, and their treatment in *alternative semantics* (Kratzer and Shimoyama, 2002; Simons, 2005a,b; Alonso-Ovalle, 2006, 2008, 2009; Aloni, 2007a,b; Menéndez-Benito, 2005, 2010, among others). In both cases, disjunction and existentials are taken to introduce sets of alternatives. In the case of alternative semantics, this treatment is motivated by a number of empirical phenomena, including free choice inferences, exclusivity implicatures, and conditionals with disjunctive antecedents. The analysis of disjunction and existentials as introducing sets of alternatives has made it possible to develop new accounts of these phenomena which improve considerably on previous accounts. However, alternative semantics does not provide any motivation for the alternative treatment of disjunction and indefinites *independently* of the linguistic phenomena at hand. Moreover, the treatment of disjunction in alternative semantics has been presented as a real *alternative* for the classical treatment of disjunction as a *join* operator. Thus, it appears that adopting the alternative treatment of disjunction forces one to give up the classical account.

The algebraically motivated inquisitive semantics presented here sheds new light on these two issues. First, it shows that, once inquisitive content is taken into consideration besides informative content, general algebraic considerations lead essentially to the treatment of disjunction that was proposed in alternative semantics, thus providing exactly the independent motivation that has so far been missing. Moreover, it shows that the 'alternative' treatment of disjunction is actually a natural generalization of the classical treatment: disjunction is still taken to behave semantically as a *join* operator, only now the meanings that this join operator applies to are more fine-grained in order to capture both informative and inquisitive content. Thus, we can have our cake and eat it: we can adopt a treatment of disjunction as introducing sets of alternatives, and still characterize it as a *join* operator.

7 Extensions

So far, we have discussed the general philosophical underpinnings of inquisitive semantics and explored in more detail the system InqB, which we consider to be the most basic implementation of the framework. In the present section, we discuss some extensions of InqB. In the extended systems, the notion of meaning is further enriched, in order to capture differences in meaning

that go beyond informative and inquisitive content, and can therefore not be captured in InqB. The extensions that we will discuss are *modular*. Thus, depending on the type of linguistic phenomenon under consideration, certain extensions may be adopted and others may be left out, in order to obtain a system that is just rich enough to deal with the phenomenon at hand.

We must note up front that the discussion in this section will be much less detailed and more speculative than in previous sections. The extensions that we will sketch are all 'work in progress', and none of them is as well-understood as the basic system InqB at this point.

7.1 Presuppositions

In section 2.3 we defined a meaning as a function f that determines, for any context s, a proposition $f(s) \in \Pi_s$, in accordance with the compatibility condition. That is, we took meanings to be *total* functions from contexts to propositions. As a consequence, in InqB sentence are always taken to express a well-defined proposition in any discourse context.

In natural language, sentences often only express a well-defined proposition in a restricted set of discourse contexts. Such sentences are said to have a *presupposition*. A sentence with a presupposition only expresses a proposition in those discourse contexts that satisfy its presupposition. Formally, this means that the meaning of a sentence with a presupposition should be modeled as a *partial* function from contexts to propositions, which is defined on a context s whenever s satisfies the presupposition. Thus, in order to incorporate presuppositions into our framework, we need to relax the totality requirement on meanings.

7.1.1 Presuppositional meanings

Presuppositions are known to be introduced by many types of constructions, such as definite descriptions (the king of France), factive verbs (know, realize), aspectual verbs (continue, stop) and adverbs (still, again), anaphoric pronouns (he, they), and temporal clauses, among others. Such constructions are called *presupposition triggers*.

We will focus here on sentences with a *factive* presupposition, that is, sentences which, in order to express a proposition, require a certain piece of information to be established in the context. The following are all examples of sentences with factive presuppositions.[6]

(4) Bea knows that John cheats on her.
$\rightarrow$ John cheats on Bea.

(5) Bea stopped smoking.
$\rightarrow$ Bea used to smoke.

(6) Bea is in Paris again.
$\rightarrow$ Bea was in Paris before.

We thus model a presupposition as a piece of information $\pi \subseteq \omega$, and define a meaning with presupposition π as a compatible function that expresses a proposition precisely in those contexts where π has indeed been established.

Definition 35 (Meanings)**.** Let π be a state. A *meaning* with a presupposition π is a function f that maps any state $s \subseteq \pi$ to a proposition $f(s) \in \Pi_s$, in accordance with the compatibility condition (see definition 14).

Notice that our former, total notion of meaning can be recovered as the particular case in which the presupposition is trivial, i.e., $\pi = \omega$. In general it will no longer be possible to identify a meaning f with the proposition it expresses in the ignorant state ω. Indeed, $f(\omega)$ need not even be well-defined. However, the compatibility condition still ensures that if $f(s)$ is well-defined and $t \subseteq s$, then $f(t)$ is completely determined by $f(s)$. As before, $f(t)$ can be obtained by restricting $f(s)$ to t in this case: $f(t) = f(s)\upharpoonright t$.

Now, if f is a meaning with presupposition π, then *any* state s on which f is defined is a subset of π, and so the proposition $f(s)$ is determined by $f(\pi)$. This means that the meaning f is jointly determined by its presupposition π and the proposition $f(\pi)$ expressed on π:

$$f(s) = \begin{cases} f(\pi)\upharpoonright s & \text{if } s \subseteq \pi \\ \text{undefined} & \text{otherwise} \end{cases}$$

Vice versa, any pair $\langle \pi, A \rangle$, where π is a state and A is a proposition over π determines a meaning f_A with presupposition π, obtained from A by restriction:

$$f_{\langle \pi, A \rangle}(s) = \begin{cases} A\upharpoonright s & \text{if } s \subseteq \pi \\ \text{undefined} & \text{otherwise} \end{cases}$$

[6]Not all presuppositions are factive. For instance, anaphoric pronouns do not presuppose that a certain piece of information has been established, but rather that a suitable antecedent has been made available.

We have thus reached the following analogue of fact 1 for the presuppositional case.

Fact 35. There is a one-to-one correspondence between meanings and pairs $\langle \pi, A \rangle$, where π is a state and A is a proposition over π.

So, now that presuppositions have been brought into the picture, we can no longer identify a meaning with a unique proposition, but we can still identify it with a unique static object, namely a pair consisting of a state (the presupposition) and a proposition over that state.

The notions of informativeness and inquisitiveness introduced in section 2.3, definition 16, still make perfect sense for presuppositional meanings: we simply call a meaning *informative* if it has the potential to provide information, and *inquisitive* if it has the potential to request information.

Definition 36 (Informativeness and inquisitiveness)**.** Let f be a meaning.

- f is *informative* if for some s, the proposition $f(s)$ is informative in s.
- f is *inquisitive* if for some s, the proposition $f(s)$ is inquisitive in s.

These properties of a meaning f can be recast in terms of properties of the associated presupposition π and proposition $f(\pi)$.

Fact 36. Let f be a meaning with presupposition π. Then:

- f is informative iff $f(\pi)$ is informative in π.
- f is inquisitive iff $f(\pi)$ is inquisitive in π.

Besides calling certain meanings informative and/or inquisitive, we will also call certain meanings *presuppositional*. The natural requirement for presuppositional meanings is of course that they have a non-trivial presupposition.

Definition 37.
We say that a meaning f with presupposition π is *presuppositional* if $\pi \neq \omega$.

7.1.2 Inquisitive semantics with presuppositions

The notion of meaning as a partial function forms the basis for an inquisitive semantics in which sentences may have presuppositions. If we consider the usual first-order language we dealt with so far, the natural thing to do is to modify the semantics to allow for partial worlds, where:

1. definite descriptions and proper names may lack a referent, i.e., a closed term t only denotes an individual in certain worlds;

2. predicates may have selectional restrictions, i.e., their interpretation in a world is a *partial* function from the domain to $\{0, 1\}$.

We can then denote by $\lfloor R(t_1, \ldots, t_n) \rfloor$ the set of worlds where all terms $t_1, \ldots, t_n$ denote individuals of the domain, and moreover the predicate associated with R is defined for these individuals, resulting in a determinate truth-value. Of course, this should all be made more precise, but since our goal is only to sketch the general features of an inquisitive semantics with presuppositions here, we do not to insist on these formal details for now.

Now, in view of fact 35, to associate a sentence φ with a certain meaning it is no longer sufficient to equip it with a proposition $[\varphi]$: we also need to specify a presupposition $\pi(\varphi) \subseteq \omega$. For the proposition $[\varphi]$, we will simply keep the clauses of InqB, as specified by definition 21.

As for presuppositions, we will, for now, adopt one of the classical accounts of presupposition projection, due to Karttunen (1974). There are of course many other accounts of presupposition projection in the literature. We do not take a stance on which of these accounts is empirically most adequate. In principle, other existing accounts can be plugged into our system as well, and the inquisitive perspective may also give rise to new approaches to the projection problem. This line of investigation, however, has not yet been pursued in much detail.

We formulate Karttunen's account by recursively defining a *presupposition satisfaction* relation $\Vdash$ between states and sentences (the clauses for the connectives are taken directly from Karttunen (1974), the others are added).

Definition 38 (Presupposition satisfaction)**.**

1. $s \Vdash R(t_1, \ldots, t_n)$ iff $s \subseteq \lfloor R(t_1, \ldots, t_n) \rfloor$

2. $s \Vdash \bot$ iff $s \subseteq \omega$

3. $s \Vdash \varphi \land \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\varphi) \Vdash \psi$

4. $s \Vdash \varphi \lor \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\neg\varphi) \Vdash \psi$

5. $s \Vdash \varphi \to \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\varphi) \Vdash \psi$

6. $s \Vdash \forall x.\varphi(x)$ iff $s \Vdash \varphi(\overline{d})$ for all $d \in D$

7. $s \Vdash \exists x.\varphi(x)$ iff $s \Vdash \varphi(\overline{d})$ for some $d \in D$

These clauses may be read as follows. The presupposition of an atomic sentence $R(t_1, \ldots, t_n)$ is satisfied in a state s just in case the terms $t_1, \ldots, t_n$ are known to denote individuals, and these individuals are known to match the selectional restrictions of the relation R. The proposition of $\bot$ is always satisfied. The presupposition of a conjunction $\varphi \land \psi$ is satisfied in a state s just in case the presupposition of φ is satisfied in s, and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\varphi)$; in other words, when evaluating the second conjunct, the information provided by the first may be assumed. The presupposition of a disjunction $\varphi \lor \psi$ is satisfied in a state s just in case the presupposition of φ is satisfied in s, and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\neg\varphi)$; so, when evaluating the second disjunct, the negation of the first disjunct may be assumed. The presupposition of an implication $\varphi \to \psi$ is satisfied in s just in case the presupposition of φ is satisfied in s and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\varphi)$; so, when evaluating the consequent of an implication, the information provided by the antecedent may be assumed. Finally, the presupposition of $\forall x \varphi(x)$ is satisfied in s just in case the presupposition of $\varphi(\overline{d})$ is satisfied in s for all elements d, and the presupposition of $\exists x \varphi(x)$ is satisfied in s just in case the presupposition of $\varphi(\overline{d})$ is satisfied in s for some element d.

Now, we define $\pi(\varphi)$ as the set of worlds that are included in some state s such that $s \Vdash \varphi$. This means that $\pi(\varphi)$ embodies the information that the actual world is located in a state that satisfies the presupposition of φ.

Definition 39 (The presupposition of a sentence)**.** $\pi(\varphi) := \bigcup\{s \mid s \Vdash \varphi\}$

Thus, for every sentence φ, we now have a way to derive the presupposition of φ, $\pi(\varphi)$, and the proposition expressed by φ, $[\varphi]$. Recall that in order for a presupposition-proposition pair $\langle \pi, A \rangle$ to determine a meaning, A should be a proposition over π. The following fact ensures that this is indeed the case for all pairs $\langle \pi(\varphi), [\varphi] \rangle$ yielded by our semantic clauses.

Fact 37 (Suitability of the semantics).
For any $\varphi \in \mathcal{L}$, $[\varphi]$ is a proposition over the state $\pi(\varphi)$.

We refer to the system defined here as InqP. Notice that in InqP, conjunction and disjunction are no longer commutative in general, since in order to determine the presupposition of a conjunction or disjunction, the order of the constituents matters.

We say that a sentence is informative (resp. inquisitive) if the associated meaning is. Using fact 35 and spelling out the definition explicitly, we obtain the following characterization.

Fact 38 (Informativeness and inquisitiveness).

- φ is informative iff $\mathsf{info}(\varphi) \subset \pi(\varphi)$
- φ is inquisitive iff $\mathsf{info}(\varphi) \notin [\varphi]$

Thus, φ is informative if it provides strictly more information than it presupposes, and inquisitive if the issue it raises is non-trivial. We can then identify assertions, questions, tautologies and hybrids in terms of informativeness and inquisitiveness, just like we did in section 4. Notice that not only assertions, but also questions, tautologies, and hybrids may have presuppositions.

7.1.3 Open and closed interrogatives

Questions are defined as non-informative sentences. By making explicit what it means to be non-informative, we get the following characterization.

Fact 39 (Explicit characterization of questions in InqP).

- φ is a question iff $\mathsf{info}(\varphi) = \pi(\varphi)$

In other words, φ is a question if and only if the proposition $[\varphi]$ forms a cover of the presupposition $\pi(\varphi)$. Now, take a generic sentence φ. There are two natural strategies to make the meaning of φ non-informative, i.e., to turn φ into a question. We may add possibilities to the proposition $[\varphi]$ in order to obtain a cover of $\pi(\varphi)$, or we may leave the proposition $[\varphi]$ untouched, and shrink the presupposition $\pi(\varphi)$ to coincide with $\mathsf{info}(\varphi)$.

The interrogative operator ? defined in section 4 as $?\varphi := \varphi \vee \neg\varphi$, implements the first strategy in the simplest possible way, adding to $[\varphi]$ the possibilities $\wp(\pi(\varphi) - \mathsf{info}(\varphi))$ to yield a cover of $\pi(\varphi)$.

But we may also implement the second strategy by expanding our language with a second interrogative operator, which we will refer to as the *closed interrogative operator*, and denote $?_c$. The semantics of this operator is specified by the following definition.

Definition 40 (Closed interrogative operator).

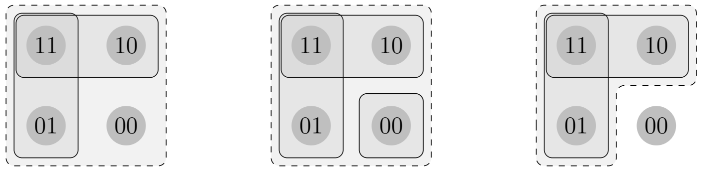
- $[?_c\varphi] = [\varphi]$
- $s \Vdash ?_c\varphi$ iff $s \subseteq \mathsf{info}(\varphi)$

To contrast the two interrogative operators, we will call the one introduced in section 4 the *open* interrogative operator, and we will denote it by $?_o$. To illustrate the behavior of the two operators, figure 10 displays their effect on a simple disjunction $P(a) \vee P(b)$, where the disjuncts are assumed for simplicity to be non-presuppositional. Figure 10(a) shows the familiar meaning of the disjunction $Pa \vee Pb$ in inquisitive semantics. As before, the maximal states in the proposition expressed by the sentence are depicted with solid borders. The presupposition of the sentence is depicted with a dashed border.

Applying the open interrogative operator we obtain the meaning depicted in figure 10(b). The resulting formula, $?_o(Pa \vee Pb)$, is a non-presuppositional question, which requests sufficient information to locate the actual world in $|Pa|$, $|Pb|$, or $|\neg(Pa \vee Pb)|$. Applying the closed interrogative operator, on the other hand, results in the meaning depicted in figure 10(c). The formula $?_c(Pa \vee Pb)$ is a question which presupposes $\mathsf{info}(Pa \vee Pb)$, i.e., it presupposes that at least one of Pa and Pb is the case, and it requests additional information in order to locate the actual world in $|Pa|$ or in $|Pb|$.

In general, a sentence φ specifies a certain set $[\varphi]$ of states. The question $?_o\varphi$ requests other participants to establish one of these states, if this is possible, or to establish a state that is incompatible with all the states in $[\varphi]$. Thus, the question $?_o\varphi$ is *open* in the sense that it leaves room for rejecting the possibilities specified by φ. The question $?_c\varphi$, on the other hand, *presupposes* that the actual world is indeed located in one of the states in $[\varphi]$. Thus, $?_c\varphi$ is *closed* in the sense that it does not leave room for rejecting the possibilities specified by φ.

Relevance for natural language semantics. The distinction between open and closed interrogative operators seems useful in analyzing the semantics of interrogatives in natural language. To illustrate this, consider (7) and (8) below, where $^{\uparrow}$ and $^{\downarrow}$ indicate rising and falling intonation, respectively.



(a) $Pa \vee Pb$ (b) $?_o(Pa \vee Pb)$ (c) $?_c(Pa \vee Pb)$

Figure 10: Open and closed interrogatives.

(7) Did you call Andrew$^{\uparrow}$, or Mark$^{\uparrow}$?

(8) Did you call Andrew$^{\uparrow}$, or Mark$^{\downarrow}$?

We may associate (7) with the open interrogative $?_o(Pa \vee Pb)$ depicted in 10(b), and (8) with the closed interrogative $?_c(Pa \vee Pb)$ depicted in 10(c). Then, the prediction is that (7), unlike (8), may be settled by a response that rejects both possibilities specified by the disjunction, such as (9).

(9) No, I didn't call Andrew or Mark.

Question (8), on the other hand, is analyzed as *presupposing* that the hearer called Andrew or Mark; thus, a response like (9) would not count as settling the issue raised by the question, but rather as going against the presupposition of the question. Another way to characterize the difference between (7) and (8) is that in uttering (8), a speaker signals that he takes the two disjuncts to provide an exhaustive list of available options, whereas this is not the case in uttering (7). Of course, this treatment extends straightforwardly to alternative questions with more than two disjuncts.

The analysis of closed alternative questions sketched here is in line with several existing accounts, both in the formal semantics literature (e.g. Rawlins, 2008; Biezma, 2009; Haida, 2010; AnderBois, 2011; Biezma and Rawlins, 2012) and in the philosophical logic literature (Hintikka, 1999, 2007; Wiśniewski, 1996, 2001; Aloni *et al.*, 2009; Aloni and Égré, 2010). Of course, there is much more to say about the meaning of such questions. For instance, they may have to be analyzed as involving an exclusive strengthening operation, like the one described in Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011). This would give rise to an analysis in which alternative questions do not just presuppose that *at least* one of the proposed possibilities

holds, but rather than *exactly* one of them holds.

Besides alternative questions, the closed interrogative operator may also be relevant for the analysis of wh-questions. We have seen in section 6 that several analyses of wh-questions may be formulated in InqB. Of course, these analyses can also be articulated in InqP. However, it has sometimes been argued that wh-interrogatives such as (10) come with an existential presupposition, of the kind expressed by (11) (see, e.g., Belnap, 1969; Keenan and Hull, 1973; Prince, 1986; Rullmann and Beck, 1998; Haida, 2007).

(10) Who did Ann go out with yesterday?

(11) Ann went out with someone yesterday.

In such analyses, answers to (10) are taken to be sentences of the form (12), where d denotes a specific individual, and the question is taken to presuppose that at least one of these answers is true.

(12) Ann went out with d yesterday.

Such approaches can be implemented straightforwardly in InqP by taking a wh-interrogative like (10) to correspond to a closed existential interrogative $?_c\exists xPx$. Indeed, the sentence $?_c\exists xPx$ is a question that presupposes that some entity has the property P, and requests sufficient information to establish for at least one entity d that it has the property P. Notice that if the domain of discourse consists of just two individuals, a and b, then $?_c\exists xPx$ is equivalent with $?_c(Pa \vee Pb)$, whose meaning is depicted in figure 10(c). Thus, under this analysis, wh-questions are a generalized form of alternative question.

This is not to say that the presuppositional analysis yields the correct account of all (or even some particular type of) wh-interrogatives. The main point here is just that such an analysis can be formulated naturally in InqP, in addition to the non-presuppositional analyses we discussed in section 6.

Finally, notice that in the case of wh-questions, just as in the case of alternative questions, one may assume that there is (in some cases) an additional exclusive strengthening operation at work; this would yield an additional uniqueness presupposition for questions like (10).

The closed interrogative operator as a projection operator. In section 4.6, we defined a non-informative projection operator as an operator $\mathcal{A}$

that turns any sentence φ into an assertion $\mathcal{A}\varphi$ having the same informative content as φ. We then argued that a non-informative projection operator could not be defined in a similar fashion, as an operator $\mathcal{Q}$ that takes any formula φ into a question $\mathcal{Q}\varphi$ having the same inquisitive content as φ. For, in InqB the inquisitive content of a question is always an issue over ω, while the inquisitive content of a generic sentence φ is an issue over $\mathsf{info}(\varphi)$, and therefore the two can only coincide if φ itself is already a question. This problem no longer arises in InqP, since the inquisitive content of a question is not required to be a cover of the whole logical space ω, but rather a cover of the presupposition $\pi(\varphi)$, which might be different from ω. Therefore, in InqP we can meaningfully adopt the following notion of a non-informative projection operator.

Definition 41 (Non-informative projection operator in InqP)**.**
An operator $\mathcal{Q}$ is a *non-informative projection operator* just in case for any φ:

- $\mathcal{Q}\varphi$ is a question
- $[\mathcal{Q}\varphi] = [\varphi]$

Now, suppose that $\mathcal{Q}$ is a non-informative projection operator, and consider any sentence φ. By definition, the proposition $[\mathcal{Q}\varphi]$ must coincide with $[\varphi]$. Moreover, since $\mathcal{Q}\varphi$ has to be a question, by fact 39 the presupposition $\pi(\mathcal{Q}\varphi)$ must coincide with $\mathsf{info}(\mathcal{Q}\varphi)$ and thus with $\mathsf{info}(\varphi)$, since $[\mathcal{Q}\varphi] = [\varphi]$. Hence, the semantics of a non-informative projection operator is uniquely determined by the above two requirements.

Fact 40 (Uniqueness of the non-informative projection operator in InqP)**.**
$\mathcal{Q}$ is a non-informative projection operator iff $[\mathcal{Q}\varphi] = [\varphi]$ and $\pi(\mathcal{Q}\varphi) = \mathsf{info}(\varphi)$.

But this is precisely the behavior that we assigned to the closed interrogative operator. Thus, $?_c$ is the non-informative operator in InqP.

Fact 41 ($?_c$ is the non-informative projection operator in InqP)**.**

- The interrogative operator $?_c$ is a non-informative projection operator.
- If $\mathcal{Q}$ is a non-informative projection operator, then $[\mathcal{Q}\varphi] = [?_c\varphi]$ and $\pi(\mathcal{Q}\varphi) = \pi(?_c\varphi)$ for all φ.

In fact, $?_c$ is in certain ways more suitably regarded as a non-informative projection operator than $?_o$. For instance, as we might expect, projecting first along one component and then along the other always lead us to the zero point of the space. In other words, trivializing first one component and then the other always results in a tautology.

Fact 42. For any sentence φ,

- $[?_c!\varphi] = [!?_c\varphi]$
- $?_c!\varphi$ and $!?_c\varphi$ are tautologies.[7]

This does not hold for $?_o$. While $!?_o\varphi$ is always a tautology, $?_o!\varphi$ may be inquisitive: for instance, $?_o!p$ is equivalent with $?_op$, which is not a tautology but rather corresponds with a polar question.

The division fact, which says that the full meaning of a sentence φ can always be reconstructed as the conjunction of the two projections, also holds for the operator $?_c$, provided that we pay attention to the ordering: φ is always equivalent with $!\varphi \wedge ?_c\varphi$, but, due to the way in which the presuppositions of a conjunction project, φ is not always equivalent with $?_c\varphi \wedge !\varphi$.

Fact 43 (Division in InqP)**.** For any φ,

$$[\varphi] = [!\varphi \wedge ?_o\varphi] = [!\varphi \wedge ?_c\varphi]$$

7.2 Impositions

The proposition expressed by a sentence φ in a state s captures the ways in which a speaker proposes to enhance that state s. Both in InqB and in InqP, the meaning of a sentence determines the proposition expressed by a sentence in a state, the only difference being that in InqP, the meaning of a sentence does not necessarily deliver a well-defined proposition for every state, thereby capturing the presupposition of the sentence. Thus, in InqP the meaning of a sentence does not only capture the ways in which a speaker proposes to enhance the common ground in uttering that sentence, but also the presuppositions that he makes concerning the current state of the common ground.

[7]Notice that tautologies are conceived of here as sentences that are neither informative nor inquisitive. They are not necessarily non-presuppositional. Indeed, $?_c!\varphi$ and $!?_c\varphi$ may very well be presuppositional.

Given this picture, it is natural to distinguish a third aspect of meaning as well. Namely, besides making certain presuppositions concerning the current state of the common ground, and proposing one or more ways to enhance the common ground, a speaker may sometimes also *impose* certain enhancements on the common ground.

The linguistic relevance of this three-way distinction between presuppositions, propositions, and *impositions* has been argued for by Murray (2010), AnderBois *et al.* (2010), and Pruitt and Roelofsen (2011).[8] In particular, it has been suggested that there is a wide range of linguistic constructions, including evidentials and appositives, whose usage typically imposes a certain enhancement on the common ground.

To illustrate this, consider the following example:

(13) Jane, who damaged her car yesterday, called Ben to ask for money.

The idea is that in uttering this sentence, a speaker presupposes that Jane has a car, imposes that she damaged it yesterday, and proposes that she called Ben to ask for money.

There are at least two empirical differences between 'what is proposed' on the one hand, and 'what is presupposed' and 'what is imposed' on the other hand (see, e.g., AnderBois *et al.*, 2010; Pruitt and Roelofsen, 2011). One difference is that an existing question under discussion is naturally addressed by what is proposed but not by what is presupposed or imposed.

(14) a. Did Jane call Ben to ask for money?
b. ✓Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

(15) a. Does Jane have a car?
b. #Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

(16) a. Did Jane damage her car yesterday?
b. #Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

Second, there is a difference, at least in English, in how a disagreeing re-

[8]The notion of impositions is also closely related to Horn's (2002) notion of *assertorically inert* implications, and to the notion of *suggestions* in (Groenendijk, 2008; Groenendijk and Roelofsen, 2009; Balogh, 2009; Roelofsen and van Gool, 2010).

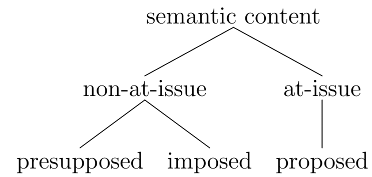


Figure 11: High-level taxonomy of semantic content.

sponse to a given utterance is marked, depending on whether it disagrees with what is proposed, or with what is presupposed or imposed. In particular, a response that disagrees with what is proposed is naturally marked with the particle *no*, while a response that disagrees with what is presupposed or imposed is more naturally marked with a weaker particle like *actually*.

(17) Jane, who damaged her car yesterday, called Ben to ask for money.
a. No, she didn't call Ben to ask for money.
b. (#No / ✓Actually), she doesn't have a car.
c. (#No / ✓Actually), she didn't damage her car yesterday.

In light of these empirical differences, we say that proposed updates constitute *at-issue* content, while presuppositions and imposed updates constitute *non-at-issue* content. This terminology is in line with much other recent literature on at-issue and non-at-issue content (e.g. Potts, 2005; Roberts *et al.*, 2009), even though the three-way distinction between presuppositions, imposed updates, and proposed updates is not generally accepted in this literature. The overall high-level taxonomy of the different types of semantic content that arises is depicted in figure 11.

A simple way to extend any of the systems considered so far, InqB, InqA, or InqP, in order to incorporate the notion of impositions would be to define meanings as functions that map a state s to a pair (i, A), where $i \subseteq s$ is an imposition over s, modeled as a set of worlds, and A is a proposition over i. Two such imposition-proposition pairs are depicted in figure 12. Impositions are represented by shapes with dashed borders, while proposition are represented by shapes with solid borders.
First consider the imposition-proposition pair depicted in figure 12(a). Let 11 be a world where Jane damaged her car yesterday and called Ben to

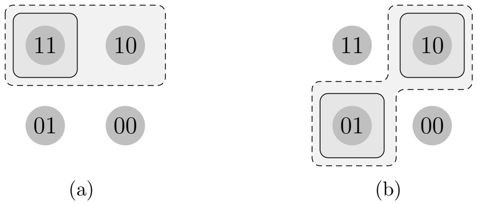


Figure 12: Two imposition-proposition pairs.

ask for money, 10 a world where Jane damaged her car yesterday but did not call Ben to ask for money, etcetera. Then the imposition-proposition pair depicted in figure 12(a) is the one expressed by our example sentence (13) in the state $s = \{11, 10, 01, 00\}$ (assuming that this state satisfies the presupposition of the sentence, i.e., that Jane has a car in all these worlds). In uttering (13) in s, a speaker imposes an update on s that restricts it to $\{11, 10\}$, i.e., worlds where Jane damaged her car yesterday, and proposes a further update that would restrict the state to $\{11\}$, i.e., the world where Jane called Ben to ask for money.

Now consider the imposition-proposition pair depicted in figure 12(b). Let 11 be a world where Jane damaged her car and her bike yesterday, 10 a world where she only damaged her car, 01 a world where she only damaged her bike, and 00 a world where she did not damage either. Then the imposition-proposition pair depicted in figure 12(b) can be taken to be the one expressed in the state $s = \{11, 10, 01, 00\}$ by the alternative question in (18) (where $\uparrow$ and $\downarrow$ represent rising and falling pitch, respectively).

(18) Did Jane damage her car$\uparrow$ or her bike$\downarrow$ yesterday?

In uttering this sentence in s, a speaker imposes an update on s which eliminates the worlds 11 and 00, leaving only those worlds where Jane damaged either her car or her bike, but not both. Moreover, the speaker proposes two further updates, eliminating either world 10 or world 01, and thus establishing whether it was Jane's car or bike that she damaged. An account along these lines of the 'exclusive component' of alternative questions has been proposed in Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011), and is also closely related to the account of Karttunen and Peters (1976).

As we have seen, the exclusive component of alternative questions can also

be treated as a presupposition, rather than an imposition, and indeed it has often been treated as such in the literature (e.g. Rawlins, 2008; Aloni *et al.*, 2009; Aloni and Égré, 2010; Biezma, 2009; AnderBois, 2010; Haida, 2010; Biezma and Rawlins, 2012). It is difficult to decide on an empirical basis which treatment is more appropriate. Strictly speaking, the presupposition account predicts that an alternative question like (18) is *uninterpretable* in a state that does not yet contain the information that Jane damaged either her car or her bike. This prediction is clearly too strong. However, it can be avoided by assuming that presuppositions may be *accommodated* in case they are not directly satisfied by the input context. With this additional assumption, however, it becomes difficult, if not impossible, to distinguish presuppositions empirically from impositions.

One theoretical option would be to reserve the notion of presuppositions for real requirements on the input context which, if not met, give rise to uninterpretability, and to re-conceptualize accommodable presuppositions as imposed updates. But we do not take a strong stance on this issue here.

Of course, once the notion of impositions is adopted, one of the main issues that arises is how they should be derived compositionally. We refer to Murray (2010), AnderBois *et al.* (2010), and Pruitt and Roelofsen (2011) for discussion of this issue, but also note that many aspects in this area are in need of further exploration.

7.3 Attentive content

In InqB, a proposition A over a state s is a non-empty, downward closed set of enhancements of s. The elements of A are states where the information provided by A has been accepted and the issue raised by A has been settled. In this way, propositions embody both informative and inquisitive content.

However, there is a natural way to further enrich this notion of propositions, in such a way that they capture even more than just informative and inquisitive content. Namely, suppose that we define a proposition A over a state s simply as a non-empty set of enhancements of s, without requiring downward closedness. Then we can still think of A as capturing informative content. Namely, as before, we could think of A as providing the information that the actual world is located in $\bigcup A$, and we could think of A as requesting sufficient information to locate the actual world inside one of the elements of A.

Thus, the elements of A are still states where the information provided by A has been accepted and the issue raised by A has been settled. However, the difference is that A no longer necessarily contains *all* such states; it may contain just some of them. This, then, allows us to think of A as capturing another aspect of meaning, which we will refer to as *attentive content*. The idea is that the states in A are enhancements of s that A draws particular attention to. They are in some sense 'priviliged' among all the states that accept the informative content and settle the inquisitive content of A.

This alternative notion of propositions gives rise to a different implementation of inquisitive semantics, which we refer to as InqA. In this setting, it is of course no longer justified to require that propositions be downward closed. After all, they may very well draw attention to a particular enhancement t of s, without drawing attention to any further enhancement t' of t. Thus, propositions are defined as arbitrary non-empty sets of enhancements.

Definition 42 (Propositions in InqA)**.**
A proposition over a state s is a non-empty set of enhancements of s.

7.3.1 Informative, inquisitive, and attentive content

As in InqB, the informative content of a proposition A is embodied by the union of all the states in A.

Definition 43 (Informative content)**.** $\mathsf{info}(A) \coloneqq \bigcup A$

The inquisitive content of a proposition A is now embodied by its *downward closure*, i.e., $A^{\downarrow} \coloneqq \{t \subseteq s \mid s \in A\}$.

Definition 44 (Inquisitive content)**.** $\mathsf{inq}(A) \coloneqq A^{\downarrow}$

In InqB, the proposition expressed by a sentence is completely determined by its informative and inquisitive content. This is not the case in InqA. To see this, consider the two propositions depicted in figure 13. In both cases, we have depicted all the states that the proposition consists of (not just the maximal states, as we did in InqB). These two propositions are clearly different, but in terms of informative and inquisitive content they are equivalent: in both cases the union of all the possibilities is $\{11, 10\}$, and the downward closure is $\{\{11, 10\}, \{11\}, \{10\}, \emptyset\}$.

The reason that the informative and inquisitive content of a sentence do not completely determine the proposition expressed by that sentence in InqA is precisely that in uttering a sentence that expresses a proposition A, a

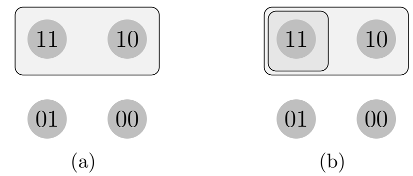


Figure 13: Two propositions in InqA that are equivalent in terms of informative and inquisitive content.

speaker is not just taken to provide the information embodied by $\mathsf{info}(A) = \bigcup A$ and raise the issue embodied by $\mathsf{inq}(A) = A^{\downarrow}$, but also to draw attention to some specific states in $A^{\downarrow}$. Thus, propositions in InqA do not only embody informative and inquisitive content, but also *attentive content*.

Definition 45 (Attentive content)**.** $\mathsf{att}(A) \coloneqq A$

7.3.2 A first-order system

Propositions in InqA may be associated with sentences in a first-order language. There are several ways in which this could be done. Below we specify one particular system, originally presented in Ciardelli (2009); Ciardelli, Groenendijk, and Roelofsen (2009).[9]

Definition 46 (A first-order system with propositions as proposals)**.**

1. $[R(t_1 \dots t_n)] \coloneqq \{|Rt_1 \dots t_n|\}$
2. $[\bot] \coloneqq \{\emptyset\}$
3. $[\varphi \wedge \psi] \coloneqq [\varphi] \sqcap [\psi]$
4. $[\varphi \vee \psi] \coloneqq [\varphi] \cup [\psi]$
5. $[\varphi \to \psi] \coloneqq \{\bigcap_{s \in A} s \Rightarrow f(s) \mid f \in [\psi]^{[\varphi]}\}$
6. $[\forall x.\varphi(x)] \coloneqq \bigsqcap_{d \in D} [\varphi(\overline{d})]$

[9]In this definition, $[\varphi] \sqcap [\psi]$ denotes the pointwise intersection of $[\varphi]$ and $[\psi]$, i.e., $\{s \cap t \mid s \in [\varphi]$ and $t \in [\psi]\}$, $s \Rightarrow f(s)$ denotes the pseudo-complement of s relative to $f(s)$, i.e., $\overline{s} \cup f(s)$, and $[\psi]^{[\varphi]}$ denotes the set of all functions from $[\varphi]$ to $[\psi]$.

7. $[\exists x.\varphi(x)] := \bigcup_{d \in D} [\varphi(\bar{d})]$

We refer to Ciardelli (2009); Ciardelli *et al.* (2009) for detailed discussion and illustration of this system, and to Westera (2012) for discussion of a closely related system, which differs from the present one in its treatment of implication. It should be noted that an algebraic motivation for a particular treatment of the logical constants in InqA is not available at this point. This is partly due to the fact that, unlike in InqB, it is not so straightforward to say when one proposition *entails* another. In InqB, propositions are compared in terms of their informative and inquisitive content. In InqA, they should also be compared in terms of their attentive content. When two propositions have exactly the same informative and inquisitive content, it is easy to say whether one is more attentive than the other. However, in other cases this is not always clear. There are several possible routes to take, but none of the ones considered so far has lead to a satisfactory algebraic motivation for a particular treatment of the logical constants in InqA.

7.3.3 Linguistic relevance

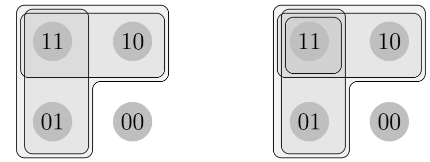
Propositions are more fine-grained in InqA than in InqB. This can be useful for several linguistic purposes. Broadly speaking we see two main advantages that InqA may have over InqB. First, it provides a more suitable basis for defining formal notions of *relatedness*, in particular answerhood and subquestionhood. And second, it allows for a new semantic perspective on what may be called *attentive operators* in natural language, i.e., operators whose semantic contribution is mainly concerned with attentive content, rather than informative or inquisitive content. Below we will say a bit more about these two potential areas of application for InqA.

Formal notions of relatedness. Consider the following contrast (see also Westera, 2012):

(19) a. Alf: Sally will bring wine or juice.
b. Bea: (Actually,) she will bring both.

(20) a. Alf: Sally will bring wine or juice, or both.
b. Bea: (*Actually,) she will bring both.

Note that Bea's response may be preceded by *actually* in (19), but not in (20). Intuitively, what *actually* seems to indicate is that the given response



(a) (19-a) (b) (20-a)

Figure 14: Possible treatment of (19-a) and (20-a) in InqA.

is possibly unexpected. It can be used by Bea in (19) because given Alf's initiative, the response is indeed possibly unexpected. In (20) on the other hand, we cannot conclude from Alf's initiative that Bea's response is possibly unexpected. Quite on the contrary, Bea's response is one of the expected responses. And therefore *actually* cannot be used in this case.

In order to turn this intuitive assessment of this particular example into a general theory of the use of *actually* (and other discourse particles / intonation patterns), we need to be able to distinguish in a systematic way between responses to a given initiative that are expected and responses that are possibly unexpected. InqB is not fine-grained enough for this purpose. In particular, since propositions are downward closed in InqB, it is impossible to assign two different propositions to the two sentences that Alf utters in (19) and (20), respectively. Thus, these two sentences come out as semantically equivalent, and there is no way of capturing the fact that Bea's response is possibly unexpected as a reaction to one but not the other.

In InqA on the other hand, the sentences that Alf utters in (19) and (20) may be assigned different propositions. The exact treatment of these sentences depends on the perspective that we take on inquisitiveness, and possibly also on the intonation pattern with which the sentences are pronounced. If we assume a strong perspective on inquisitiveness and an intonation pattern which indicates that Alf invites, but does request an informative response in uttering (19-a) and (20-a), then these sentences may be associated with the propositions depicted in figure 14.[10]

[10]These propositions may be derived compositionally by assuming that the declarative complementizer functions as a non-inquisitive projection operator !, whose semantic contribution in InqA is defined as follows: $[!\varphi] := [\varphi] \cup \{\bigcup[\varphi]\}$. See Ciardelli, Groenendijk, and Roelofsen (2010).

Once our semantics is fine-grained enough to distinguish between (19-a) and (20-a), we are in a position to try to capture the distinction between expected and possibly unexpected responses to a given sentence. One hypothesis, for instance, would be that ψ is an expected response to φ if $[\varphi] \subseteq [\psi]$, and that it is a possibly unexpected response otherwise. This hypothesis would correctly predict that (19-b) is a possibly unexpected response to (19-a) but not to (20-a).

This is just to illustrate that it is possible in InqA to characterize certain notions of relatedness, in this case a particular notion of answerhood, that are impossible to capture in InqB. Even if the hypothesis formulated here is too simplistic, the framework allows for alternative, more sophisticated hypotheses as well. And besides the particular notion of answerhood illustrated here, other notions of relatedness may of course be of interest as well.

Attentive operators. A second area of application is the analysis of operators in natural language whose semantic contribution is mainly concerned with attentive content, rather than informative or inquisitive content. In Ciardelli *et al.* (2009, 2010) it is argued that English *might*, at least on some of its usages, can be seen as such an operator. Usually, *might* is analyzed as an epistemic possibility modal. However, it is well-known that *might* interacts with the propositional connectives in peculiar ways. In particular, it behaves differently in this respect from expressions like 'it is possible that' or 'it is consistent with my beliefs that', which is problematic for any account that analyzes *might* as an epistemic modal. Its analysis as an attentive operator sheds new light on this issue.

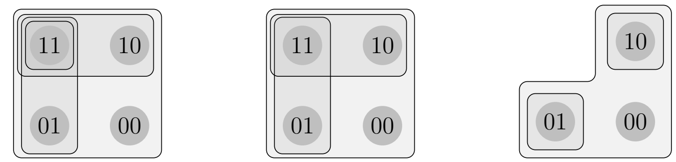
Let us illustrate this with some concrete examples. Consider the sentences in (21), (22), and (23). In order to deal with such sentences we enrich our logical language with an operator, $\Diamond$, which is intended to correspond to *might* in English. With the addition of this operator, each English sentence in (21)–(23) has a straightforward translation into our logical language, which is given to its right.

(21) John might be in Paris or in London. $\Diamond(p \vee q)$

(22) John might be in Paris or he might be in London. $\Diamond p \vee \Diamond q$

(23) John might be in Paris and he might be in London. $\Diamond p \wedge \Diamond q$

Zimmermann (2000, p.258–259) observed that (21), (22), and (23) are all



(a) $\Diamond p \wedge \Diamond q$

(b) $\Diamond p \vee \Diamond q$
$\approx \Diamond(p \vee q)$

(c) $\Diamond p \wedge \Diamond q$
$\approx \Diamond p \vee \Diamond q$
$\approx \Diamond(p \vee q)$

Figure 15: *Might* interacting with conjunction and disjunction.

equivalent.[11] This is not the case for similar sentences with clear-cut epistemic modalities. For instance, (24) is clearly not equivalent with (25).

(24) It is consistent with my beliefs that John is in London or it is consistent with my beliefs that he is in Paris.

(25) It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

This contrast is problematic for modal accounts of *might*. A further subtlety is that Zimmermann's observation seems to crucially rely on the fact that 'being in London' and 'being in Paris' are mutually exclusive. To see this, consider the following examples:

(26) John might speak English or French. $\Diamond(p \vee q)$

(27) John might speak English or he might speak French. $\Diamond p \vee \Diamond q$

(28) John might speak English and he might speak French. $\Diamond p \wedge \Diamond q$

'Speaking English' and 'speaking French' are not mutually exclusive, unlike 'being in London' and 'being in Paris'. As a result, the equivalence partly breaks down: (26) and (27) are still equivalent with each other, but not with (28). To see this, consider a situation, suggested to us by Anna Szabolcsi, in which someone is looking for an English-French translator, i.e.,

[11]These type of examples have also often been discussed in the recent literature in relation to the phenomenon of *free choice permission*, which involves deontic modals (cf. Geurts, 2005; Simons, 2005a; Alonso-Ovalle, 2006; Aloni, 2007b; Fox, 2007; Klinedinst, 2007; Chemla, 2009).

someone who speaks *both* English and French. In that context, (28) would be perceived as a useful recommendation, while (26) and (27) would not.

These patterns can be accounted for quite straightforwardly if *might* is treated as an operator that trivializes the informative and inquisitive content of its complement, but preserves its attentive content. This is achieved by the following treatment of $\Diamond$ (Ciardelli *et al.*, 2009, 2010).

Definition 47 (Might in InqA)**.**

- $[\Diamond\varphi] \coloneqq [\varphi] \cup \{\omega\}$

For any φ, the proposition expressed by $\Diamond\varphi$ consists of all states in $[\varphi]$ plus the 'trivial' state ω. This means that the proposition expressed by $\Diamond\varphi$ relative to a particular state s consists of all states in $[\varphi]_s$ plus the state s itself. Thus, in uttering $\Diamond\varphi$ in s, a speaker proposes exactly the same enhancements of s that he would have proposed in uttering φ, with the addition of the trivial enhancement, which amounts to leaving s unchanged.

Notice that for any φ, $\mathsf{info}(\Diamond\varphi) = \omega$ and $\mathsf{inq}(\Diamond\varphi) = \wp(\omega)$. This means that $\Diamond\varphi$ is never informative or inquisitive. Thus, $\Diamond$ indeed trivializes the informative and inquisitive content of its complement, while preserving the attentive content.

Now let us return to the examples above. The proposition expressed by $\Diamond p \wedge \Diamond q$ is depicted in figure 15(a), and the proposition expressed by $\Diamond(p \vee q)$ and $\Diamond p \vee \Diamond q$ is depicted in figure 15(b). Notice that $\Diamond p \wedge \Diamond q$, unlike $\Diamond(p \vee q)$ and $\Diamond p \vee \Diamond q$, draws attention to the state $\{11\}$, which embodies the information that John speaks both English and French. This explains the observation that (28) is perceived as a useful recommendation in the translator-situation, unlike (26) and (27).

In Zimmermann's original example, p stands for 'John is in London' and q for 'John is in Paris'. It is impossible for John to be both in London and in Paris, so in dealing with this particular example, we should assume a logical space that does not contain worlds where p and q are both true, i.e., a logical space consisting of the worlds 10, 01, and 00, but not 11. Relative to this logical space, $\Diamond(p \wedge q)$, $\Diamond p \vee \Diamond q$, and $\Diamond p \wedge \Diamond q$ all express exactly the same proposition, as depicted in figure 15(c). Thus, the intuition that Zimmermann's original examples are all equivalent is also accounted for.

There is of course much more to say about the treatment of *might* as an attentive operator, about the relation between this account and the modal account, as well as the dynamic account of Veltman (1996), and about other

operators in natural language that may be treated (partially) as attentive operators. We refer to Ciardelli *et al.* (2009, 2010) for further discussion.

7.4 Highlighting

Evidently, one of the empirical domains where inquisitive semantics is intended to be put to use is the semantics of *interrogatives*. We emphasized in section 6 that inquisitive semantics should not be taken to constitute a particular *theory* of interrogatives, but rather a general framework in which several such theories may be formulated and compared. We mentioned in particular the classical theories of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) as ones that may be formulated in inquisitive semantics.

However, there is also a number of existing approaches to the semantics of interrogatives that cannot be articulated in inquisitive semantics, at least not in the most basic implementation of the framework. This holds in particular for the structured meaning approach of von Stechow (1991) and Krifka (2001), the dynamic approach of Aloni and van Rooij (2002); Aloni *et al.* (2007), and the orthoalgebraic approach of Blutner (2012).

Proponents of these approaches have explicitly argued against the classical 'proposition set' approach of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984). One of the main arguments is based on a contrast in the interpretation of polarity particle responses to 'opposing' polar interrogatives (see especially Krifka, 2001; Blutner, 2012):

(29) Is the door open?
 a. Yes $\Rightarrow$ the door is open
 b. No $\Rightarrow$ the door is closed

(30) Is the door closed?
 a. Yes $\Rightarrow$ the door is closed
 b. No $\Rightarrow$ the door is open

In terms of inquisitive content, (29) and (30) are entirely equivalent: each of these polar interrogatives elicits a choice between two possibilities, the possibility that the door is open, and the possibility that the door is closed.[12] However, there is a clear difference between the two interrogatives in terms

[12] It is assumed here that *open* and *closed* are antonyms, but nothing hinges on this assumption—parallel examples could be constructed with, e.g., *even* and *odd*.

of polarity particle responses: in response to (29), *yes* means that the door is open, and *no* means that the door is closed, whereas in response to (30), *yes* means that the door is closed, and *no* means that the door is open.[13]

A further contrast is noted in Roelofsen and van Gool (2010): while (31) below is also equivalent with (29) and (30) in terms of inquisitive content, it does not license polarity particle responses at all:

(31) Is the door open or closed?
 a. #Yes.
 b. #No.

A semantic account of this contrast is beyond the reach of theories that are formulated in InqB, or in any of the more fine-grained systems considered so far. However, in our view these observations do not constitute an argument against the framework in general. Rather, they show that there is an aspect of meaning that the systems considered so far do not yet capture.

In particular, while these systems do suitably capture informative and inquisitive potential, they do not yet capture the *anaphoric potential* of sentences, i.e., the potential to set up discourse referents that may serve as antecedents for subsequent anaphoric expressions.

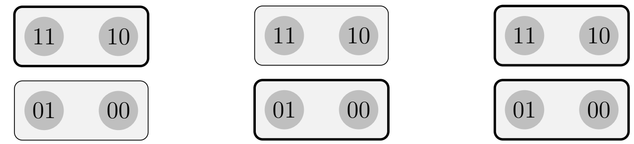
To overcome this limitation, and more specifically to account for the contrast in (29)–(31), Roelofsen and van Gool (2010), Pruitt and Roelofsen (2011), and Farkas and Roelofsen (2012) develop an extension of InqA in which a sentence may *highlight* some of the possibilities that make up the proposition that it expresses. When a sentence is uttered, these highlighted possibilities become available as antecedents for subsequent anaphoric expressions. Polarity particles, then, can be analyzed as such anaphoric expressions: they either confirm or reject the highlighted antecedent possibilities.

Intuitively, the possibilities that are highlighted are the ones that are *explicitly mentioned*. The idea is that, in virtue of being mentioned explicitly, these possibilities are made more salient than other possibilities, and therefore more readily accessible for subsequent anaphoric elements.[14]

Consider for instance the polar question in (29). There is an intuitive

[13]It should be noted that Groenendijk and Stokhof (1984, pp.321–323) actually provide an explicit account of the interpretation of *yes* and *no* that captures the difference between (29) and (30).

[14]This formulation should of course not be taken all too literally—strictly speaking, possibilities do not get 'mentioned' by expressions in the object-language.



(a) ⟦(29)⟧ (b) ⟦(30)⟧ (c) ⟦(31)⟧

Figure 16: Highlighted and non-highlighted possibilities for (29)–(31).

sense in which the possibility that the door is open is explicitly mentioned by this question, while the possibility that the door is closed is not. So (29) highlights the possibility that the door is open, while (30) highlights the possibility that the door is closed, and (31) highlights both possibilities.[15] This is depicted in figure 16, where 11 and 10 are worlds where the door is open, 01 and 00 are worlds where the door is closed, and highlighted possibilities are displayed with a thick border.

Now, if we assume that *yes* presupposes a unique highlighted possibility, and confirms this possibility in case the presupposition is met, while *no* presupposes one or more highlighted possibilities, and rejects all of these possibilities in case its presupposition is met, we obtain a straightforward account of the observed contrast in (29)–(31). In the case of (29), there is exactly one highlighted possibility, so both *yes* and *no* are licensed: *yes* confirms the highlighted possibility, conveying that the door is open, while *no* rejects the highlighted possibility, conveying that the door is closed. In the case of (30), there is again exactly one highlighted possibility, only now this is the possibility that the door is closed. So, again, both *yes* and *no* are licensed, only now *yes* conveys that the door is closed, while *no* conveys that the door is open. Finally, in the case of (31) there are two highlighted possibilities. This means that *yes* is not licensed because its presupposition fails, while *no* is contradictory, since the two highlighted possibilities together cover the entire logical space.

For further details and extension of the account of polarity particles sketched here, as well as further motivation for the notion of highlighting, we refer to Pruitt and Roelofsen (2011) and Farkas and Roelofsen (2012).

[15]For details on how highlighted possibilities are computed compositionally we refer to Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011); Farkas and Roelofsen (2012).

8 Inquisitive pragmatics and discourse

In this section we outline the role that inquisitive semantics is intended to play in an overall theory of interpretation and information flow in discourse. In section 8.1 we discuss how inquisitive semantics, in enriching the basic notion of semantic meaning, also gives rise to a richer perspective on pragmatics. In section 8.2 we show how inquisitive semantics can be integrated with dynamic epistemic logic (van Ditmarsch *et al.*, 2007; van Benthem, 2011, among others), obtaining a formal framework which does not only allow us to explicitly model the semantic interpretation of sentences, but also the flow of information that takes place when these sentences are uttered in discourse.

8.1 Inquisitive pragmatics

The main objective of Gricean pragmatics (Grice, 1975, and much subsequent work) is to explain aspects of interpretation which are not directly dictated by semantic content, in terms of general features of rational human behaviour. Since inquisitive semantics enriches the basic notion of semantic content, it gives rise to a new perspective on pragmatics as well.

The Gricean maxims specify what it means for the participants of a conversation to behave rationally. However, the theory as it has been developed so far has two important limitations. First, it is exclusively *speaker-oriented*, and second, it is only concerned with what it means for speakers to behave rationally in *providing* information, and not, for instance, in requesting information.

To illustrate this point, consider Grice's Quality and Quantity maxims. The Quality maxim says that a speaker should only provide information that is supported by his own information state, and the Quantity maxim says that a speaker should provide as much information as possible, as long as the information is relevant for the current purposes of the conversation. Clearly, both maxims are speaker-oriented, and only concerned with what it means to behave rationally in providing information.

Inquisitive semantics gives rise to a pragmatics which is both speaker-oriented and hearer-oriented, and which is not only concerned with what it means to behave rationally in providing information, but more generally with what it means to behave rationally in *exchanging* information.

What it means for a particular participant to behave rationally in exchanging information partly depends, of course, on the overall goals of that

participant. For instance, her intention may be to obtain a certain piece of information from other participants, while concealing other pieces of information that she herself already has access to. We will focus here, however, on the case in which all participants try to resolve a given issue as effectively as possible in a fully cooperative way. Below we discuss two qualitative requirements that all participants should adhere to in such a cooperative effort, *sincerity* and *transparency*. For discussion of quantitative preferences and a formal notion of relevance that play a role in this setting, we refer to Groenendijk and Roelofsen (2009).

Sincerity. First of all, participants must be *sincere*. This requirement is comparable to Grice's Quality maxim. However, in the present setting, several types of sincerity can be distinguished. First, if a speaker utters a sentence φ, she must believe that the actual world is located in at least one of the states in $[\varphi]$. This means that the speaker's information state should be contained in $\mathsf{info}(\varphi)$. We refer to this requirement as *informative sincerity*.

Second, if φ is inquisitive relative to the common ground, then it should also be inquisitive relative to the speaker's own information state. Otherwise, the speaker would be raising an issue that she could just as well have settled herself. We refer to this requirement as *inquisitive sincerity*.

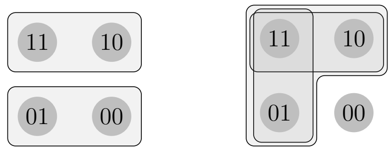
And third, if a speaker draws attention to a particular enhancement of the common ground, then that enhancement should be compatible with her information state. Formally, this means that the proposition that φ expresses in InqA, which captures the attentive content of φ, should only contain states that have at least one world in common with the speaker's information state. We refer to this requirement as *attentive sincerity*.

To illustrate these requirements and the implicatures that they give rise to, consider the following examples.

(32) Does John speak French? $?p$

(33) John speaks English or French. $!(p \vee q)$

The propositions that we take these sentences to express in InqA are depicted in figure 17. First consider (32). Suppose that a speaker S with information state σ utters this sentence, and suppose that no common information has been established in the conversation yet, which means that the common ground amounts to ω. First note that S cannot fail to be informatively sincere in uttering (32), since the informative content of the sentence is trivial. Next,



(a) $[(32)]$ (b) $[(33)]$

Figure 17: The propositions expressed by (32) and (33) in InqA.

note that (32) is inquisitive in the given common ground. Thus, in order for S to be inquisitively sincere, (32) should also be inquisitive in σ. This means that both states in $[(32)]$ should overlap with σ. Thus, we derive as sincerity implicatures that S should consider it possible that John speaks French and that S should consider it possible that John does not speak French. Notice that this is also precisely what is needed for S to be attentively sincere in uttering (32). So the attentive sincerity requirement does not give rise to additional implicatures in this case. Notice that the ignorance implicature that arises here is inherently linked to inquisitiveness and attentiveness, and cannot be derived straightforwardly from the standard Gricean maxims.

Next, consider (33), which we translate into our formal language as $!(p \vee q)$. Again, suppose that a speaker S with information state σ utters this sentence, and suppose that the common ground amounts to ω. In this case, the informative sincerity requirement is not trivially met. Rather, σ should be contained in $\mathsf{info}(!(p \vee q))$, which means that S should believe that John speaks English or French. Second, note that $!(p \vee q)$ is not inquisitive in the given common ground (or in any other state for that matter), which means that the inquisitive sincerity requirement is trivially satisfied. However, the attentive sincerity requirement is only satisfied if every state in $[!(p \vee q)]$ has a non-empty overlap with σ. Thus, in this case we derive as sincerity implicatures that S should consider it possible that John speaks English and that S should consider it possible that John speaks French.

We should note that the attentive sincerity requirement is sometimes outweighed by efficiency considerations. To see this, consider the sentences in (34):

(34) a. John speaks a European language.
b. Which European languages does John speak?

Presumably, the propositions expressed by these two sentences both contain the state consisting of all worlds where John speaks French. However, intuitively, a speaker who utters these sentence does not implicate that she considers it possible that John speaks French. Strictly speaking, a speaker who utters these sentences knowing that John does not speak French, violates the attentive sincerity requirement. However, there is a tradeoff between sincerity and efficiency in these cases. For instance, instead of uttering (34-b), the speaker may explicitly list all the European languages of which she considers it possible that John speaks them, but this is likely to be a less efficient move (depending on how many such languages there are). Thus, in uttering sentences like (34-a-b), a speaker does implicate that she does not know exactly which European languages John speaks, but she does not implicate that for every European language she does not know whether John speaks that language.

Notice that this tradeoff does not exist in the case of (32) and (33). More generally, the attentive sincerity requirement may be outweighed by efficiency considerations in the case of indefinites and *wh*-interrogatives, but not in the case of disjunctions and polar interrogatives.

Transparency. Sincerity is a speaker-oriented qualitative requirement. Its hearer-oriented counterpart is *transparency*. If a speaker draws attention to a particular enhancement of the common ground which is inconsistent with the hearer's information state, then the hearer must publicly signal this inconsistency, to make sure that the enhancement is indeed not established. On the other hand, if one participant makes a certain proposal and no other participant objects, then each participant must incorporate the informative content of the proposal into her own information state and into her representation of the common ground.

To illustrate this requirement consider the disjunctive statement in (33) above. Suppose that this sentence is uttered and that one of the participants knows that John does not speak English. Then, in order to satisfy the transparency requirement, she should publicly announce that John does not speak English, even if she does not know whether or not John speaks French. On the other hand, if none of the participants objects to the proposal that is made in uttering (33), then all participants should update their own information state and their representation of the common ground with the information that John speaks English or French.

Sincerity, transparency, and attentive *might*. To end this subsection, we will briefly consider the basic repercussions of the qualitative sincerity and transparency requirements just discussed for the treatment of *might* as an attentive operator sketched in section 7.3.

There are two basic empirical observations concerning *might* that we did not discuss at all in section 7.3, even though each of them has given rise to one of the two 'classical' semantic theories of *might*. Both observations can be illustrated by means of the following example:

(35) John might speak English.

The first observation is that if someone utters (35) we typically conclude that she considers it possible that John speaks English. This observation has given rise to the classical analysis of *might* as an epistemic modal operator.

The second observation is that if someone hears (35) and already knows that John does not speak English, she will typically object, pointing out that (35) is inconsistent with her information state. In this sense, even though *might* sentences do not provide any information about the state of the world, they can be 'inconsistent' with a hearer's information state. The standard account of this observation is that of Veltman (1996). Veltman's update semantics specifies for any given information state σ and any given sentence φ, what the information state $\sigma[\varphi]$ is that would result from updating σ with φ. The update effect of $\Diamond\varphi$ is defined as follows:

$$\sigma[\Diamond\varphi] = \begin{cases} \emptyset & \text{if } \varphi \text{ is inconsistent with } \sigma \\ \sigma & \text{otherwise} \end{cases}$$

The idea is that, if φ is inconsistent with a hearer's information state, then updating with $\Diamond\varphi$ leads to the absurd state. To avoid this, the hearer must make a public announcement signaling the inconsistency of φ with her information state. As a result, the participant who uttered $\Diamond\varphi$ in the first place may also come to discard the possibility that φ holds.

Our semantic treatment of *might* as an attentive operator does not directly explain these two observations. However, both observations can be explained pragmatically. On the one hand, it follows from the attentive sincerity requirement that a cooperative speaker who utters (35) must consider it possible that John speaks English. On the other hand, it follows from the transparency requirement that if a hearer is confronted with (35), and one of the possibilities for φ is inconsistent with her information state, then she must signal this inconsistency.

Thus, both observations are accounted for. And this pragmatic account, unlike the semantic analyses just mentioned, extends straightforwardly to more involved cases. Consider for instance:

(36) John might speak English or French.

This sentence is problematic for both semantic accounts. The epistemic modality account predicts that the speaker considers it possible that John speaks English or French. But note that this is compatible with the speaker knowing perfectly well that John does not speak English. What (36) implies is something stronger, namely that the speaker considers it possible that John speaks English *and* that she considers it possible that John speaks French. This follows straightforwardly on our pragmatic account.

Now consider a hearer who is confronted with (36) and who knows that John possibly speaks French, but certainly not English. We expect this hearer to object to (36). But Veltman's update semantics does not predict this: it predicts that an update with (36) has no effect on the hearer's information state. Our pragmatic account on the other hand, does urge the hearer to object.

The only task of our semantics is to specify the proposition expressed by each sentence, and thus the proposal that would be made in uttering that sentence. The pragmatics, then, specifies what a context—in particular, the common ground and the information state of the speaker—must be like in order for a certain proposal to be made, and how a hearer is supposed to react to a given proposal, depending on the common ground and her own information state. In the case of *might* sentences, shifting some of the weight from semantics to pragmatics evades problems with more involved cases, like (36), in a straightforward way. But, of course, the necessary pragmatic principles can only be stated if the underlying semantics captures more than just informative content.

For further discussion of the interplay of semantics and pragmatics in the interpretation of *might* sentences, especially in embedded contexts, we refer to Ciardelli *et al.* (2009, 2010); Roelofsen (2011b).

8.2 Modeling inquisitive discourse

Inquisitive semantics provides a framework which allows us to formally capture the semantic interpretation of a sentence, and we have just sketched how certain further pragmatic inferences may be accounted for as well. However, eventually all this should be embedded in a formal framework that does not only allow us to specify the semantic interpretation of a sentence and derive further pragmatic inferences, but which also allows us to explicitly model the flow of information that results from uttering a sentence in discourse.

Such a framework is provided by dynamic epistemic logic (DEL) (van Ditmarsch *et al.*, 2007; van Benthem, 2011, among others), a system that is designed to model explicitly how utterances change the epistemic states of the participants in a conversation. Dynamic epistemic logic builds on ordinary (static) epistemic logic (EL) (Hintikka, 1962), which in turn is an extension of classical propositional logic (CPL). Below we provide a brief overview of epistemic logic and dynamic epistemic logic, and then indicate how inquisitive semantics can be integrated with these formalisms, drawing on Roelofsen (2011c) and Ciardelli (2012).

8.2.1 Epistemic logic

The language of EL is obtained from the language of CPL by adding a knowledge operator K_a for each relevant agent a.[16] Intuitively, $K_a\varphi$ means that agent a knows that φ holds. The semantics of EL is given in terms of Kripke models. A Kripke model for a set of agents $\mathcal{A}$ and a set of proposition letters $\mathcal{P}$ is a tuple:

$$\langle W, w_0, \{R_a \mid a \in \mathcal{A}\}, V\rangle$$

where W is a set of possible worlds, $w_0 \in W$ is a possible world representing the actual world, $R_a \subseteq W \times W$ is a binary relation on W representing the epistemic state of agent a, and $V : \mathcal{P} \to \wp(W)$ is a valuation function which assigns to every atomic sentence a set of worlds in W, i.e., the set of worlds in which that sentence is true.

Intuitively, $\langle w, v\rangle \in R_a$ means that in world w, agent a considers world v possible. A sentence $K_a\varphi$ is true in a world w just in case φ holds in all worlds that a considers possible in w, i.e., all worlds v such that $\langle w, v\rangle \in R_a$.

For any relation $R \subseteq W \times W$, we will write $R[w]$ for the set of all worlds that stand in the relation R to w, $\{v \mid \langle w, v\rangle \in R\}$. The clause for knowledge operators can then be formulated as follows:

Definition 48 (Interpretation of knowledge operators in EL)**.**

[16] The term *knowledge* is used here as a placeholder; many subtly different notions of knowledge and belief can in fact be modeled in epistemic logic.

- $M, w \models K_a\varphi$ iff for all $v \in R_a[w]$, $M, v \models \varphi$

Kripke models do not only capture the agents' knowledge about the configuration of the world, but also their knowledge about other agents' knowledge, and about those other agents' knowledge about yet other agents' knowledge, etcetera. In short, epistemic states in Kripke models embody *higher-order information*. Moreover, epistemic logic is not only concerned with the knowledge of individual agents, but also with various notions of group knowledge. In particular, the logical language is often further extended with an operator C, which stands for *common knowledge*. Intuitively, $C\varphi$ means that it is common knowledge among all agents that φ holds, i.e., everyone knows that φ holds, and everyone knows that everyone knows that φ holds, etcetera. Formally, $C\varphi$ is true in a world w just in case φ holds in any world v such that $\langle w, v\rangle$ is in the transitive closure of $\bigcup_{a\in\mathcal{A}} R_a$, which we will denote as R^*_M.

Definition 49 (Interpretation of the common knowledge operator in EL)**.**

- $M, w \models C\varphi$ iff for all $v \in R^*_M[w]$, $M, v \models \varphi$

8.2.2 Dynamic epistemic logic

DEL extends EL in such a way that it becomes possible to formally specify and reason about the effects of certain types of speech acts on the epistemic states of the conversational participants. Most work in the DEL tradition has so far focused on one particular type of speech act, namely that of making an *assertion*. To model the effect of an assertion, the logical language of EL is expanded with expressions of the form $[!\varphi]_a\psi$. Intuitively, $[!\varphi]_a\psi$ means that an assertion of φ by agent a leads to a state where ψ holds. Formally, $[!\varphi]_a\psi$ is defined to be true in a world w in a model M just in case ψ is true in w in the model $M^{!_a\varphi}$, which is obtained from M by means of a procedure that captures the effect of an assertion of φ by agent a. There are several ways to define $M^{!_a\varphi}$. For concreteness, we adopt the following definition, in which we use $R_{a,\varphi}$ to denote the relation $\{\langle w, v\rangle \mid \langle w, v\rangle \in R \text{ and } M, v \models \varphi\}$.

Definition 50 (The effect of an assertion in DEL)**.**

- $M^{!\varphi_a} = \begin{cases} \langle W, w_0, \{R_{a,\varphi}\} \cup \{R_b \mid b \in A\}, V\rangle & \text{if } M, w_0 \models K_a\varphi \\ \text{undefined} & \text{otherwise} \end{cases}$

According to this definition, an assertion of φ by a is only felicitous if in the actual world a knows that φ is the case. If this condition is met, then the effect of the assertion is to make $K_a\varphi$ common knowledge. Otherwise, the effect of the assertion is undefined.

Fact 44. For any M, w such that $M, w_0 \models K_a\varphi$: $M^{!\varphi_a}, w \models CK_a\varphi$

Now that we know how assertions change the model of evaluation, we can specify precisely how sentences of the form $[!\varphi]_a\psi$ are interpreted.

Definition 51 (Interpretation of assertion operators in DEL)**.**

- $M, w \models [!\varphi]_a\psi$ iff $M^{!\varphi_a}, w \models \psi$

Notice that the basic DEL system presented so far is truth-conditional. This means that in this system the semantic meaning of a sentence is identified with its informative content. Moreover, the speech act of making an assertion is completely characterized by the information that is provided in performing that speech act. Thus, inquisitiveness is not yet part of the picture, neither at the level of semantic content, nor at the level of speech acts. Evidently, inquisitiveness does play a crucial role in the process of exchanging information. This has been recognized in recent work within the DEL tradition, in particular by Van Benthem and Minică (2011). In order to bring inquisitiveness into the picture, Van Benthem and Minică enrich the logical language with a second dynamic speech act operator, $[?\varphi]_a$. This operator is used to describe the effects of a speech act of asking whether φ holds, performed by agent a. Intuitively, $[?\varphi]_a\psi$ means that the speech act of asking whether φ holds, performed by agent a, leads to a state where ψ holds. We will refer to the resulting system as DELQ.

A crucial feature of DELQ is that the basic static fragment of the logical language (which can be thought of as the language that the agents in the conversation speak) does not contain any sentences that are interrogative in any syntactic sense, or sentences that are inquisitive in any semantic sense. A question is seen as a specific kind of *speech act* that may be performed by an agent. But in terms of syntactic form and semantic content, sentences that are used in asking questions are not taken to be any different from sentences that are used in making assertions. In particular, they are not interrogative or inquisitive in any sense.

An alternative approach would be to actually enrich the semantics of the basic static fragment of the logical language, in such a way that the proposition expressed by every sentence in this fragment already captures both its informative and its inquisitive content. This enrichment is precisely what is provided by inquisitive semantics. On this alternative approach, the static fragment of the language could be taken to contain interrogative sentences of the form $?\varphi$, and such sentences could be taken to express inquisitive propositions, embodying the issue of whether φ is the case. The dynamic part of the language could then be simplified: instead of having separate assertion and question operators, $[!\varphi]_a$ and $[?\varphi]_a$, we could have a single *utterance operator* $[\varphi]_a$, where φ could be syntactically indicative or interrogative, and semantically informative and/or inquisitive. Intuitively, $[\varphi]_a\psi$ would then mean that an utterance of φ by agent a leads to a state where ψ holds. Thus, on this approach, inquisitiveness does not enter the picture at the speech act level, but rather already at the level of the syntax and semantics of the basic static language.

In Roelofsen (2011c) it is argued that this alternative approach has some crucial advantages. Most importantly, it makes it possible to deal with *embedded* questions, such as conditional questions (e.g., *If John goes to the party, will Mary go as well?*) and questions embedded under knowledge operators (e.g. *John knows whether Mary will go*). This is impossible if questions only enter the picture at the speech act level, because in such a setup the logical language does not contain sentences of the form $p \to ?q$ or $K_a?q$.

This second approach gives rise to an *inquisitive epistemic logic* (IEL) and an *inquisitive dynamic epistemic logic* (IDEL). Below we specify and discuss the main features of these systems. For further motivation and discussion we refer to Roelofsen (2011c) and Ciardelli (2012).

8.2.3 Inquisitive epistemic logic

We will start by presenting the system IEL, an inquisitive semantics for the language of epistemic logic, with ! and ? as additional operators. As in InqB, we define $!\varphi$ as an abbreviation of $\neg\neg\varphi$ and $?\varphi$ as an abbreviation of $\varphi \vee \neg\varphi$.

The semantics of IEL is stated in terms of Kripke models, just like the semantics of EL. The only difference is that now, sentences are not evaluated relative to worlds, but relative to *states*, which are sets of worlds, as in InqB. We formulate the semantics of IEL in terms of support (see section 4.7).

Definition 52 (Support)**.** Let M be a model and s a state.

1. $M, s \models p$ iff $s \subseteq V(p)$

2. $M, s \models \bot$ iff $s = \emptyset$

3. $M, s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

4. $M, s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$

5. $M, s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$

6. $M, s \models K_a\varphi$ iff for all $w \in s$, $R_a[w] \models \varphi$

7. $M, s \models C\varphi$ iff for all $w \in s$, $R^*_M[w] \models \varphi$

As in InqB, support is persistent, i.e., if $M, s \models \varphi$ and $t \subseteq s$, then $M, t \models \varphi$ as well. As usual in inquisitive semantics, support is persistent. The atomic clause and the clauses for the Boolean connectives are just as in InqB. The clause for individual knowledge operators says that $K_a\varphi$ is supported by a state s just in case for every world $w \in s$, φ is supported by the epistemic state of agent a in w. The clause for common knowledge says that $C\varphi$ is supported in a state s just in case for every $w \in s$, $R^*_M[w]$ supports φ. Notice that as a result of this definition, $K_a\varphi$ and $C\varphi$ are never inquisitive, regardless what φ is.

Example 1. IEL provides a unified treatment of knowledge-*that* and knowledge-*whether* constructions, assuming that *that* is translated into our logical language as !, and *whether* as ?. Consider the following sentences:

(37) a. Alex knows that Peter is coming. $K_a!p$
b. Alex knows that Peter or Quinten is coming. $K_a!(p \vee q)$
c. Alex knows whether Peter is coming. $K_a?p$

For a state s to support the first sentence, every w in s must be such that $R_a[w]$ supports $!p$, which means that every world in $R_a[w]$ must be one where p holds. Similarly, for s to support the second sentence, every w in s must be such that $R_a[w]$ supports $!(p \vee q)$, which means that every world in $R_a[w]$ must be one where either p or q holds. Finally, for s to support the third sentence, every w in s must be such that $R_a[w]$ supports $?p$, which means that we must either have that every world in $R_a[w]$ is one where p holds, or that every world in $R_a[w]$ is one where $\neg p$ holds. These are precisely the

desired predictions for these sentences.[17]

We define the proposition $[\varphi]_M$ expressed by φ in M as the set of all states supporting φ, and the informative content of φ in M as $|\varphi|_M := \bigcup[\varphi]_M$. We could also define informative and inquisitive sentences, as well as questions, assertions, and hybrids, exactly as we did in InqB. Finally, the notion of entailment also directly carries over from InqB to IEL.

Thus, the extension of InqB to the language of epistemic logic is rather straightforward. The next step is to add dynamic speech act operators to the system.

8.2.4 Inquisitive dynamic epistemic logic

Recall that in DELQ there are two speech act operators, one for assertions and one for questions. This is necessary because in DELQ the proposition expressed by a sentence only embodies the informative content of that sentence. In IEL, the proposition expressed by a sentence captures both its informative and its inquisitive content. This means that we no longer need to introduce two distinct speech act operators for questions and assertions. Instead we can have a single operator for utterances more generally. In addition to this, we will introduce an *acceptance* operator, which is used to model the speech act of accepting the informative content of a previously uttered sentence.

Thus, the language of IDEL is obtained from the language of IEL by adding expressions of the form $[\varphi]_a\psi$ and $[ok]_a\psi$. Intuitively, $[\varphi]_a\psi$ means that an utterance of φ by agent a leads to a state that supports ψ, while $[ok]_a\psi$ means that acceptance by agent a of the informative content of the previously uttered sentence leads to a state that supports ψ.

Speech acts are taken to change the discourse context. We have seen that in DEL, the discourse context is represented by a Kripke model, which captures the epistemic states of all the conversational participants. In order to capture the effect of an acceptance speech act, this simple notion of a discourse context needs to be extended somewhat. In particular, we need to keep track of the sentences that have been uttered so far, since the effect of an acceptance speech act depends on the informative content of the previously

[17] The present system may be further refined in order to account for embedded disjunctive questions (incorporating ideas from Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011) and to deal with one of Gettier's famous puzzles concerning the notion of knowledge as justified true belief, which involves sentences with disjunctive clauses embedded under knowledge operators (see Uegaki, 2011).

uttered sentence. Thus, we define a discourse context X as a pair $\langle M, T\rangle$, where M is a Kripke model and T is a stack of sentences, i.e., those sentences that have been uttered so far. Following Farkas and Bruce (2010) and Farkas and Roelofsen (2011), we refer to T as the Table of the conversation.

Definition 53 (Stacks)**.**

- For any $n \in \mathbb{N}$, a stack of length n is a tuple with n elements.
- If T is a stack of length $n \geq 1$, then for every $0 \leq m \leq n$, T_m denotes the mth element of T.
- If T is a stack of length $n \geq 1$, then top(T) denotes the nth element of T.
- If T is a stack of length n, and x an object, then $T + x$ is a stack T' of length $n+1$, such that $T'_m = T_m$ for all $1 \leq m \leq n$, and $T'_{n+1} = x$.

Definition 54 (Discourse contexts)**.** A discourse context is a pair $\langle M, T\rangle$, where M is a Kripke model and T a stack of sentences.

Now we are ready to specify the effect of an utterance on the discourse context. We take the effect of an utterance of φ by an agent a to be twofold: first, φ is put on the Table, and second, the epistemic state of agent a in every world in the current model is restricted to the informative content of φ. Thus, it becomes common knowledge that a's epistemic state supports the informative content of φ. This idea is captured by the following definition, where we use $R_{a,\varphi}$ to denote the relation $\{\langle w, v\rangle \mid \langle w, v\rangle \in R \text{ and } v \in |\varphi|_M\}$.

Definition 55 (The effect of an utterance on the discourse context)**.**
Let $X = \langle M, T\rangle$ be a discourse context, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\mathsf{IDEL}}$. Then:

$$X^{\varphi_a} = \langle M^{\varphi_a}, T^{\varphi_a}\rangle$$

where:

1. $M^{\varphi_a} = \begin{cases} \langle W, w_0, \{R_{a,\varphi}\} \cup \{R_b \mid b \in A\}, V\rangle & \text{if } M, w_0 \models K_a\varphi \\ \text{undefined} & \text{otherwise} \end{cases}$
2. $T^{\varphi_a} = T + \varphi$

The speech act of acceptance has a simpler effect than that of uttering a sentence: it does not put a new proposal on the Table, but only eliminates worlds in which the epistemic state of the agent of the speech act does not support the informative content of the proposition that is on top of the Table. Thus, in making an acceptance move, a speaker publicly commits to the informative content of the previously uttered sentence.

Definition 56 (The effect of acceptance on the discourse context)**.**
Let $\langle M, T\rangle$ be a discourse context, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\mathsf{IDEL}}$. Then:

$$X^{ok_a} = \langle M^{ok_a}, T^{ok_a}\rangle$$

where:

1. $M^{ok_a} = \begin{cases} \langle W, w_0, \{R_{a,\mathsf{top}(T)}\} \cup \{R_b \mid b \in A\}, V\rangle & \text{if } T \neq \emptyset \\ \text{undefined} & \text{otherwise} \end{cases}$
2. $T^{ok_a} = T$

Notice that $\langle M, T\rangle^{ok_a}$ is only well-defined if T contains at least one element. This reflects the anaphoric nature of acceptance: an acceptance move is appropriate only if there is at least one previously uttered sentence on the Table.

Having specified how utterances and acceptance moves affect the discourse context, we are now ready to define when a state supports a sentence in IDEL, given a certain discourse context. The first seven clauses are essentially the same as those for IEL. The two additional clauses deal with constructions involving speech act operators.

Definition 57 (Support in IDEL)**.**
Let $X = \langle M, T\rangle$ be a discourse context, $p \in \mathcal{P}$, $a \in \mathcal{A}$, and $\varphi, \psi \in \mathcal{L}_{\mathsf{IDEL}}$.

1. $X, s \models p$ iff $s \subseteq V(p)$
2. $X, s \models \bot$ iff $s = \emptyset$
3. $X, s \models \varphi \wedge \psi$ iff $X, s \models \varphi$ and $X, s \models \psi$
4. $X, s \models \varphi \vee \psi$ iff $X, s \models \varphi$ or $X, s \models \psi$
5. $X, s \models \varphi \rightarrow \psi$ iff for all $s' \subseteq s$: if $X, s' \models \varphi$ then $X, s' \models \psi$

6. $X, s \models K_a\varphi$ iff for all $w \in s : X, R_a[w] \models \varphi$
7. $X, s \models C\varphi$ iff for all $w \in s : X, R^*_M[w] \models \varphi$
8. $X, s \models [\varphi]_a\psi$ iff $X^{\varphi_a}, s \models \psi$
9. $X, s \models [ok]_a\psi$ iff $X^{ok_a}, s \models \psi$

The proposition expressed by a sentence φ in a discourse context X can be defined as the set of all states that support φ in X. All the other by now familiar notions, such as the informative content of a sentence, informative and inquisitive sentences, questions, assertions, hybrids, and entailment carry over straightforwardly from InqB via IEL to IDEL.

Moreover, several discourse related notions can be defined in IDEL. For instance, we could say that a discourse context $\langle M, T\rangle$ is *stable* if and only if none of the sentences in T are inquisitive in M. Intuitively, this means that all the issues that have been raised so far are settled. Similarly, we could say that a sentence φ has the potential to *resolve* a discourse context $\langle M, T\rangle$ just in case an utterance of φ by one of the agents, and subsequent acceptance by all the other agents, would lead to a stable discourse context. We could also add operators corresponding to these notions to the object language. For instance, we could add an operator R to the language, and say that a sentence $R\varphi$ is supported by s relative to a discourse context X if and only if φ has the potential to resolve X. A detailed exploration of such notions will be left for another occasion.

For now, we note one particular feature of the system, which seems especially natural and desirable. Namely, if a sentence φ is uttered by an agent a and subsequently accepted by all other agents $b \neq a$, its informative content cannot fail to become common knowledge.

Fact 45. If $\mathcal{A} = \{a_1, \dots, a_n\}$, then for any discourse context X, any state s, and any sentence φ:

$$X, s \models [\varphi]_{a_1}[ok]_{a_2} \dots [ok]_{a_n} C!\varphi$$

Finally, we would like to mention two possible ways to further enhance the system discussed here. First, it would be possible to enrich Kripke models in such a way that they do not just associate an information state with every agent in every world, but rather an information state plus an issue over that state, representing not just the information that the agent currently has, but

also the information that she would like to acquire. This approach is explored in Ciardelli (2012).

Second, Kripke models may be enriched in such a way that they do not only capture the agents' knowledge, but also their *discourse commitments*. This would make it possible to model situations in which an agent commits to a certain piece of information without really knowing whether this piece of information actually holds. In this setting, it is also natural to go one step further, and to introduce *conditional commitments*. It is natural to think of certain speech acts as involving such conditional commitments. Consider for instance the tag-question in (38):

(38) John is coming, isn't he?

Arguably, in uttering (38) a speaker conditionally commits to the possibility that John is coming, i.e., on the condition that the responder commits to this possibility, the speaker commits to it as well. An approach along these lines is explored in Farkas and Roelofsen (2012).

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