

Category Theory for All: Homework 1

Valeria de Paiva, valeria.depaiva@gmail.com

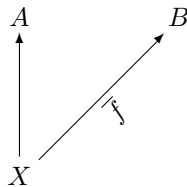
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Part I: Generic questions about you

1. Which degree are you pursuing, if any?
2. Do you have favorite and/or less liked parts of Mathematics? What are they? Why? (don't worry if you don't know that, I can never say why I don't like the bits of Maths that I don't like..)
3. Would you prefer to answer #2, if Mathematics was substituted by Programming/Linguistics/Philosophy/etc...? Which would be your favorite and less liked bits then?

Part II: Categories

1. Consider usual sets with partial functions. A partial function f from A to B is a total function from a subset of A , say X , to B which we could write as:



Draw and define the natural composition of two partial morphisms and show that sets and partial maps form a category PartFun.

2. Now consider a collection of structures Pno (you will show it's a category..) whose objects are triples (A, α, a) where A is a set, a is an element of A and $\alpha: A \rightarrow A$ is a chosen map. Given two such structures (A, α, a) and (B, β, b) a morphism $f: (A, \alpha, a) \rightarrow (B, \beta, b)$ is a function such that $\alpha; f = f; \beta$ and $f(a) = b$.

A. Verify that Pno is a category.

B. Show that $(N, succ, 0)$ where $succ$ is the usual successor function on the natural numbers is an object of Pno.

C. Show that for any object (A, α, a) of Pno there is a unique arrow $f: (N, succ, 0) \rightarrow (A, \alpha, a)$ and describe the behaviour of the function f .