Category Theory for All: Homework 1

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Part I: Generic questions about you

1. Which degree are you pursuing, if any?

2. Do you have favorite and/or less liked parts of Mathematics? What are they? Why? (don’t worry if you don’t know that, I can never say why I don’t like the bits of Maths that I don’t like..)

3. Would you prefer to answer #2, if Mathematics was substituted by Programming/Linguistics/Philosophy/etc...? Which would be your favorite and less liked bits then?

Part II: Categories

1. Consider usual sets with partial functions. A partial function \( f \) from \( A \) to \( B \) is a total function from a subset of \( A \), say \( X \), to \( B \) which we could write as:

\[
\begin{array}{ccc}
A & \rightarrow & B \\
\downarrow & & \\
X & & \\
\end{array}
\]

Draw and define the natural composition of two partial morphisms and show that sets and partial maps form a category \( \text{PartFun} \).

2. Now consider a collection of structures \( \text{Pno} \) (you will show it’s a category..) whose objects are triples \((A, \alpha, a)\) where \( A \) is a set, \( a \) is an element of \( A \) and \( \alpha: A \rightarrow A \) is a chosen map. Given two such structures \((A, \alpha, a)\) and \((B, \beta, b)\) a morphism \( f: (A, \alpha, a) \rightarrow (B, \beta, b) \) is a function such that \( \alpha; f = f; \beta \) and \( f(a) = b \).

A. Verify that \( \text{Pno} \) is a category.

B. Show that \((N, \text{succ}, 0)\) where \( \text{succ} \) is the usual successor function on the natural numbers is an object of \( \text{Pno} \).

C. Show that for any object \((A, \alpha, a)\) of \( \text{Pno} \) there is an unique arrow \( f: (N, \text{succ}, 0) \rightarrow (B, \beta, b) \) and describe the behaviour ofthe function \( f \).