Category Theory for All: Homework 1

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Part I: Generic questions about you

1. Which degree are you pursuing, if any?
2. Do you have favorite and/or less liked parts of Mathematics? What are they? Why? (don't worry if you don't know that, I can never say why I don't like the bits of Maths that I don't like..)
3. Would you prefer to answer #2, if Mathematics was substituted by Programming/Linguistics/Philosophy/etc...?Which would be your favorite and less liked bits then?

Part II: Categories

1. Consider usual sets with partial functions. A partial function f from A to B is a total function from a subset of A, say X, to B which we could write as:

$$\begin{tikzcd} A & & B \\ & & \\ X \arrow[uu] \arrow[uurr, "f"'] & & \end{tikzcd}$$

   Draw and define the natural composition of two partial morphisms and show that sets and partial maps form a category $\mathsf{PartFun}$.

2. Now consider a collection of structures Pno (you will show it's a category..) whose objects are triples (A, α, a) where A is a set, a is an element of A and $\alpha\colon A \to A$ is a chosen map. Given two such structures (A, α, a) and (B, β, b) a morphism $f\colon (A, \alpha, a) \to (B, \beta, b)$ is a function such that $\alpha; f = f; \beta$ and $f(a) = b$.

   A.Verify that Pno is a category.

   B.Show that $(N, succ, 0)$ where $succ$ is the usual succesor function on the natural numbers is an object of Pno.

   C. Show that for any object (A, α, a) of Pno there is an unique arrow $f\colon (N, succ, 0) \to (B, \beta, b)$ and describe the behaviour ofthe function f.