

# Category Theory for All: Homework 1

Valeria de Paiva, valeria.depaiva@gmail.com

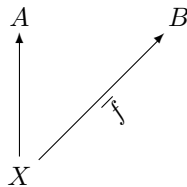
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## Part I: Generic questions about you

1. Which degree are you pursuing, if any?
2. Do you have favorite and/or less liked parts of Mathematics? What are they? Why? (don't worry if you don't know that, I can never say why I don't like the bits of Maths that I don't like..)
3. Would you prefer to answer #2, if Mathematics was substituted by Programming/Linguistics/Philosophy/etc...? Which would be your favorite and less liked bits then?

## Part II: Categories

1. Consider usual sets with partial functions. A partial function  $f$  from  $A$  to  $B$  is a total function from a subset of  $A$ , say  $X$ , to  $B$  which we could write as:



Draw and define the natural composition of two partial morphisms and show that sets and partial maps form a category PartFun.

2. Now consider a collection of structures Pno (you will show it's a category..) whose objects are triples  $(A, \alpha, a)$  where  $A$  is a set,  $a$  is an element of  $A$  and  $\alpha: A \rightarrow A$  is a chosen map. Given two such structures  $(A, \alpha, a)$  and  $(B, \beta, b)$  a morphism  $f: (A, \alpha, a) \rightarrow (B, \beta, b)$  is a function such that  $\alpha; f = f; \beta$  and  $f(a) = b$ .

A. Verify that Pno is a category.

B. Show that  $(N, succ, 0)$  where  $succ$  is the usual successor function on the natural numbers is an object of Pno.

C. Show that for any object  $(A, \alpha, a)$  of Pno there is a unique arrow  $f: (N, succ, 0) \rightarrow (A, \alpha, a)$  and describe the behaviour of the function  $f$ .