A note on the complexity of backward induction games

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RAIN @ NASSLLI 2012

university of groningen

Outline

Introduction

Computational complexity

Complexity of a single trial

Outlook

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Logic and CogSci?

Question

What can logic do for CogSci, and vice versa?

Marr's levels of explanation

1. computational level:
   - problems that a cognitive ability has to overcome

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1. computational level:
   - problems that a cognitive ability has to overcome
2. algorithmic level:
   - the algorithms that may be used to achieve a solution
3. implementation level:
   - how this is actually done in neural activity

Marr, Vision: a computational investigation into the human representation and processing of the visual information, 1983

Between computational and algorithmic level

Claim

Logic can inform us about inherent properties of the problem.

Level 1,5 Complexity level:

- complexity of the possible algorithms

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Example

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The shorter the proof the easier the problem.

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Example

The easier the algorithm the easier quantifier verification.

Szymanik & Zajenkowski, Comprehension of simple quantifiers, 2010
More: 13.45 @ TLS

Logic and social cognition

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1. Higher-order reasonings: ‘I believe that Ann knows that Ben thinks . . . ’

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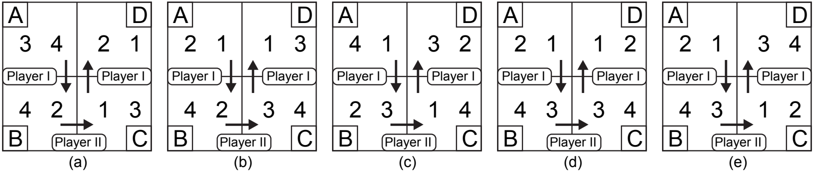
Logic and social cognition

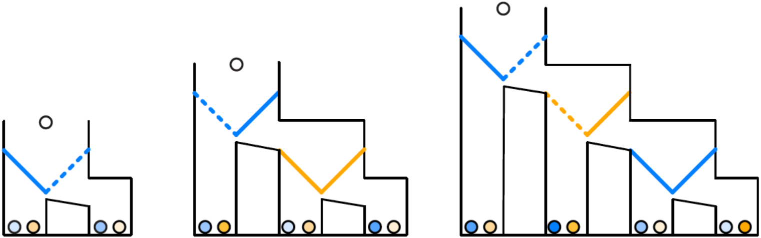
1. Higher-order reasonings: 'I believe that Ann knows that Ben thinks . . . '
2. Interacts with game-theory
3. Backward induction: tells us which sequence of actions will be chosen by agents that want to maximize their own payoffs, assuming common knowledge of rationality.
4. BI games have been extensively studied in psychology

Matrix game



Hedden & Zhang What do you think I think you think?, 2002

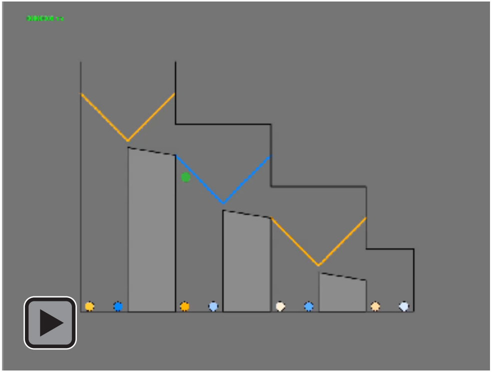
Marble Drop Game



(a) (b) (c)

Meijering et al., The facilitative effect of context on second-order social reasoning, 2010

MDG performance . . .



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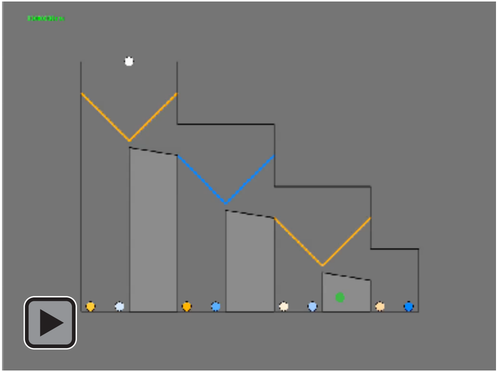
MDG performance gets better



BI algorithm

At the end of the game, players have their values marked. At the intermediate stages, once all follow-up stages are marked, the player to move gets her maximal value that she can reach, while the other, non-active player gets his value in that stage.

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$\hookrightarrow$ human reasoning strategies

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BI is computable in polynomial time

- Breadth-first search.

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- Breadth-first search.
- Therefore, $BI \in PTIME$.

Question

Is BI PTIME-complete?

Question

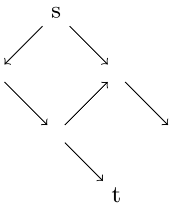
Descriptive complexity analysis of BI?

Van Benthem & Gheerbrant, Game solution, epistemic dynamics and fixed-point logics, 2010

Preliminaries: reachability

Question

Is t reachable from s?



Preliminaries: reachability

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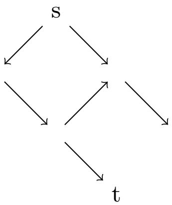


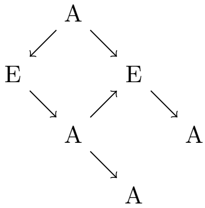
Theorem

Reachability is NL-complete.

Alternating graphs

Definition

Let an alternating graph $G = (V, E, A, s, t)$ be a directed graph whose vertices, V, are labeled universal or existential. $A \subseteq V$ is the set of universal vertices. $E \subseteq V \times V$ is the edge relation.



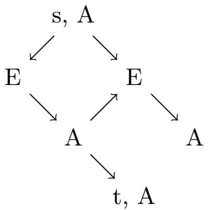
Reachability on alternation graphs

Definition

Let $G = (V, E, A, s, t)$ be an alternating graph. We say that t is reachable from s iff $P_a^G(s,t)$, where $P_a^G(x,y)$ is the smallest relation on vertices of G satisfying:

1. $P_a^G(x,x)$
2. If x is existential and $P_a^G(z,y)$ holds for some edge (x,z) then $P_a^G(x,y)$.
3. If x is universal, there is at least one edge leaving x, and $P_a^G(z,y)$ holds for all edges (x,z) then $P_a^G(x,y)$.

Is there an alternating path from s to t?



Reachability on alternating graphs is PTIME-complete

Definition

$REACH_a = \{G | P_a^G(s,t)\}$

Theorem

$REACH_a$ is PTIME-complete via first-order reductions.

Corollary on competitive games

Observation

Given G and s, $REACH_a$ intuitively corresponds to the question:
'Is s a winning position for the first player in the competitive game G?'

Corollary

BI for competitive games is PTIME-complete.

Extensive form game graphs

Definition

A two player game $G = (V, E, V_1, V_2, f_1, f_2, s, t)$ is a graph, where V is the set of nodes, $E \subseteq V \times V$ is the edge relation (available moves). For $i = 1, 2$, $V_i \subseteq V$ is the set of nodes controlled by Player i, and $V_1 \cap V_2 = \emptyset$. Finally, $f_i : V \longrightarrow \mathbb{N}$ assigns pay-offs for Player i.

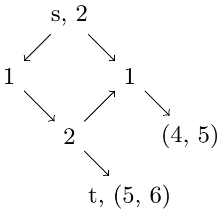
BI accessibility relation

Definition

Let G be a two player game. We define the backward induction accessibility relation on G. Let $P_{bi}^{G}(x, y)$ be the smallest relation on vertices of G such that:

1. $P_{bi}^{G}(x, x)$
2. Take $i = 1, 2$. Assume that $x \in V_i$ and $P_{bi}^{G}(z, y)$. If the following two conditions hold, then also $P_{bi}^{G}(x, y)$ holds:
   2.1 $E(x, z)$;
   2.2 there is no w, v such that $E(x, w)$, $P_{bi}^{G}(w, v)$, and $f_i(v) > f_i(y)$.

And now, is s Bi-accessible from t?



BI decision problem

Definition

$REACH_{bi} = \{G | P^G_{bi}(s,t)\}$

Theorem

$REACH_{bi}$ is PTIME-complete via first-order reductions.

Is it interesting?

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- Difficult to effectively parallelize.
- Difficult to solve in limited space.

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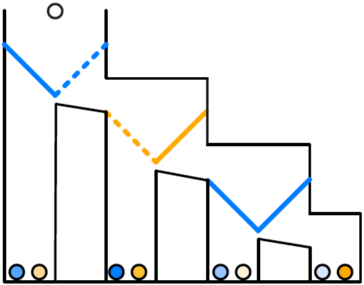
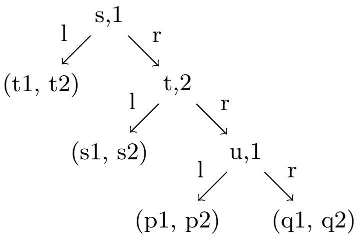


MDG decision trees



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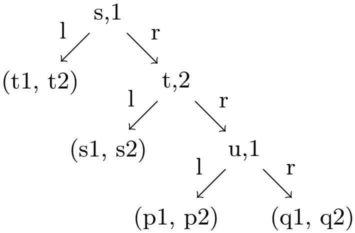

Definition

G is generic, if for each player, distinct end nodes have different pay-offs.

Question

Question

How to approximate the complexity of a single instance?

Alternation type

Definition

Let's assume that the players strictly alternate in the game. Then:

1. In a Λ_1^i tree all the nodes are controlled by Player i.
2. In a Λ_k^i tree, k-alternations, starts with an ith Player node.

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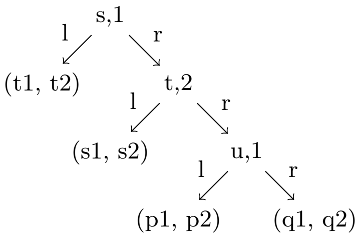

Figure: Λ_3^1 -tree

Alternation hierarchy

Conjecture

For every $i, j \in \{1, 2\}$, *the computational complexity of* $REACH_a$ *for all* Λ^i_{n+1} *graphs is greater than for all* Λ^j_n *graphs, and all* Λ^i_n *graphs are of the same complexity.*

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$\hookrightarrow$ corresponds to logarithmic hierarchy for the competitive case

Subjects strategies

To explain eye-tracking data: forward induction with backward reasoning

Subjects strategies

To explain eye-tracking data: forward induction with backward reasoning

Definition

If T is a generic game tree with the root node controlled by Player 1 (2) and n is the highest pay-off for Player 1 (2), then T^- is the minimal subtree of T containing the root node and the node with pay-off n for Player 1 (2).

Λ_3^1 trees

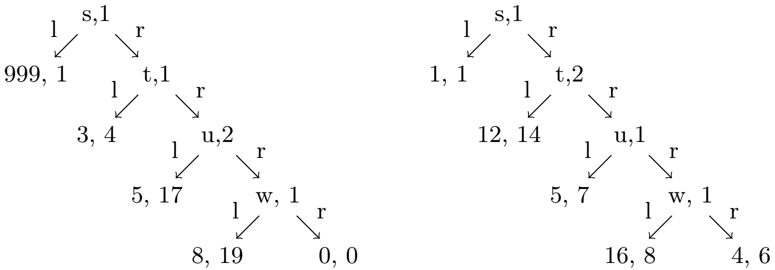

Figure: Two Λ_3^1 trees.

T^{-}-example

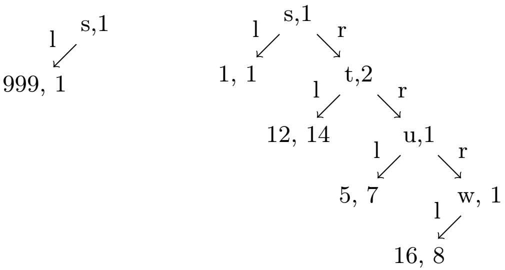

Figure: Λ_1^1 tree and Λ_3^1 tree

Alternations × pay-offs

Conjecture

Let us take two MDG trials T_1 and T_2. T_1 is easier than T_2 if and only if T_1^- is lower in the tree alternation hierarchy than T_2^-.

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$\hookrightarrow$ to be checked with the eye-tracking data

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Logic

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- Describing agents' internal reasoning.

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- Define modal/alternation depth of formulas.

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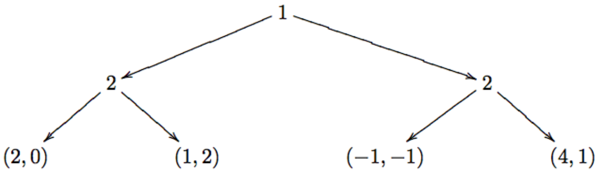
- Build proof-system.
- Define proof-depth that corresponds to the reasoning difficulty.

Thanks!

Example



A proof:

1. $turn_2 \wedge \langle 2 \rangle (u_2 = 0 \wedge u_1 = 2) \wedge \langle 2 \rangle (u_2 = 2 \wedge u_1 = 1) \wedge (2 > 1)$ (premise)
2. $turn_2 \wedge \langle 2 \rangle (u_2 = -1 \wedge u_1 = -1) \wedge \langle 2 \rangle (u_2 = 1 \wedge u_1 = 4) \wedge (2 > 1)$ (premise)
3. $(u_2 = 2 \wedge u_1 = 1)$ (from 1)
4. $(u_2 = 1 \wedge u_1 = 4)$ (from 2)
5. $(u_1 = 1 \wedge u_2 = 2)$ (from 3)
6. $(u_1 = 4 \wedge u_2 = 1)$ (from 4)
7. $turn_1 \wedge \langle 1 \rangle (u_1 = 1 \wedge u_2 = 2) \wedge \langle 2 \rangle ((u_1 = 4 \wedge u_2 = 1) \wedge (4 > 1)$ (from 5, 6)
8. $(u_1 = 4 \wedge u_2 = 1)$ (from 2) (from 7)