Deontic Paradoxes from a Dynamic Perspective

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Reasoning and Interaction at NASSLLI

June 23, 2012

1 The Plot

- Key claim: combining insights from dynamic semantics with recent work on the semantics of deontic modals delivers attractive solutions to a number of paradoxes about deontic conditionals. Here I focus on the the notorious case from Forrester (1984).
- (1) and (2) are jointly consistent and do not exclude the possibility that Jones in fact violates the obligation not to murder Smith.

  (1) Jones ought not murder Smith.

  (2) If Jones murders Smith, he ought to murder Smith gently.

- But suppose that:

  (3) Jones murders Smith.

  (2) and (3) entail:

  (4) Jones ought to murder Smith gently,

  which contradicts (1). So (1)-(3) are not consistent after all. Where did we go wrong?

- There are lots of solutions out there. Here my aim is to demonstrate that we can give a solution to the paradox satisfying the following desiderata:
  - Conditional obligations detach: (4) follows from (2) under the supposition that (3) is true. This criterion excludes responses to the paradox denying that conditional obligations detach.[1]
  - (4) indeed contradicts (1). This criterion excludes solutions that either reject closure of *ought* under logical entailment or try to exploit subtle issues about tense or the logical form of (4) to explain away the contradiction.[2]

[1] Authors who reject detachment for conditional obligations in one form or another include, among others, Åqvist (1984), Belnap et al. (2001), Kolodny and MacFarlane (2010), Lewis (1974), and Pietroski (1993).

[2] Forrester (1984) rejects closure of *ought* under logical entailment. Castañeda (1985, 1986) and Sinnott-Armstrong (1985) appeal to ambiguities in the structure of deontically modalized sentences to avoid a contradiction between (1) and (4). See Goble (1991) for a critical discussion, who endorses Forrester's solution to the paradox. Temporal machinery plays a major role in the treatment of deontic conditionals by Loewer and Belzer (1983, 1986), Sellars (1967), and Thomason (1981a,b), among many others.

  - The response to the paradox develops from an independently plausible semantics for conditionals and deontic modals. This criterion excludes solutions to the paradox that use a primitive binary conditional obligation operator.[3]

- Key idea: there is no paradox since at no point in the argument are we committed to *both* (1) and (4). That is, nowhere in the argument do we endorse the *conjunction* of "Jones ought not murder Smith" and "Jones ought to murder Smith gently." This is just to deny that the right logic for *ifs* and *oughts* is monotonic in the following sense:

  Monotonicity If $\phi_1, \ldots, \phi_n \vDash \psi$, then $\phi_1, \ldots, \phi_n, \phi_{n+1} \vDash \psi$

2 The Framework

2.1 Semantics

- Following Kolodny and MacFarlane (2010) I treat deontic modals as informational modals. Let w be a possible world, i be an information state (a set of possible worlds):

$$\llbracket \Box_f \phi \rrbracket^{w,i} \text{ is true iff for all } w' \in f(i),\ \llbracket \phi \rrbracket^{w',i} \text{ is true}$$
$$\llbracket \Diamond_f \phi \rrbracket^{w,i} \text{ is true iff for some } w' \in f(i),\ \llbracket \phi \rrbracket^{w',i} \text{ is true}$$

  Here f is a selection function mapping information states to modal quantifier domains. Deontic *ought* is treated as a necessity operator whose domain is selected by a deontic selection function d. We impose two constraints on the deontic selection function:

  - **Non-Emptiness** For all $i \neq \varnothing$, $d(i) \neq \varnothing$
  - **Realism** For all i, $d(i) \subseteq i$

- To prepare the semantics for conditionals, define a process of strengthening:

  > The result of *strengthening* i with ϕ, $i + \phi$, is defined as the intersection of i and $\llbracket \phi \rrbracket^{i}$, i.e. $i + \phi = i \cap \{w\colon \llbracket \phi \rrbracket^{w,i} \text{ is true}\}$

  The semantics for conditionals is then defined as follows (inspired by Kratzer (1991)):

$$\llbracket \phi \Rightarrow \psi \rrbracket^{w,i} \text{ is true iff } \llbracket \psi \rrbracket^{w,i+\phi} \text{ is true}$$

  Here the assumption is that conditional consequents contain a modal operator. By default, the modal is a (perhaps implicit) epistemic necessity operator but in iffy oughts the modal is deontic.

- Observe that this semantics for *ifs* and *oughts* immediately entails failure of what Gillies (2010) calls *t-persistence*:

  > ϕ is *t-persistent* iff for all w, i and $i' \subseteq i$: if $\llbracket \phi \rrbracket^{w,i}$ is true then $\llbracket \phi \rrbracket^{w,i'}$ is true

  "Jones ought not murder Smith" is true given i but false (or at least not true) given i as strengthened with the information that Jones murders Smith.

[3] The use of primitive binary conditional obligation operators in deontic logic is a popular tradition starting with the work of von Wright (1956), Rescher (1958, 1962), and Chisholm (1964).

2.2 Logical Consequence

- Kolodny and MacFarlane propose the following notion of logical consequence:

  $\phi_1, \ldots, \phi_n \vDash_{K\&M} \psi$ iff for all w and i such that $w \in i$: if $\langle w, i \rangle \in [\![\phi_1]\!]$ and...and $\langle w, i \rangle \in [\![\phi_n]\!]$, then $\langle w, i \rangle \in [\![\psi]\!]$

  This does not suit my purposes since it is monotonic and denies that conditional obligations detach.

- A dynamic alternative to Kolodny's and MacFarlane's static proposal (see also Gillies (2009)):

  $\phi_1, \ldots, \phi_n \vDash_D \psi$ iff for all w and i such that $w \in i$: if $w \in [\![\phi_1]\!]^i$ and... and $w \in [\![\phi_n]\!]^{i+\ldots+\phi_{n-1}}$, then $w \in [\![\psi]\!]^{(i+\ldots+\phi_{n-1})+\phi_n}$

  Information strengthening plays two distinct roles. First, adding another premise to an argument has the classical effect of restricting the set of possible worlds at which its conclusion is to be evaluated. Second, and since we now require that truth at a world is information sensitive, adding another premise to the argument strengthens the informational parameter with respect to which subsequent claims are evaluated.

- Dynamic logical consequence has no commitment to monotonicity since we have lack of t-persistence: additional information may fail to preserve truth at a point of evaluation.

2.3 Problem Solved

- Supplementing the semantics for *ifs* and *oughts* with a dynamic conception of logical consequence resolves our paradox in accordance with the three desiderata articulated above:
  - Conditional obligations detach since modus ponens is dynamically valid: $\phi \Rightarrow \psi, \phi \vDash_D \phi$
  - (1) and (4) are jointly inconsistent
  - The proposal is based on an independently plausible semantics for *ifs* and *ought*

- We avoid a contradiction due to the nonmonotonicity of dynamic logical consequence:
  - Let Γ consist of (1) and (2), i.e. $\Gamma = \{\Box_d \neg m, m \Rightarrow \Box_d g\}$. While $\Gamma \vDash_D \Box_d \neg m$, $\Gamma, m \nvDash_D \Box_d \neg m$. So even though $\Gamma, m \vDash_D \Box_d g$ and $\Box_d \neg m \wedge \Box_d g \vDash_D \bot$, $\Gamma, m \nvDash_D \bot$ and thus $\Gamma \nvDash_D \neg m$
  - In words, nowhere in the argument are we committed to both "Jones ought not murder Smith" and "Jones ought to murder Smith gently." Specifically, the assumption that Jones murders Smith that licenses the inference to "Jones ought to murder Smith gently" defeats our commitment to "Jones ought not murder Smith."
  - All this, keep in mind, not by fiat but because of the interaction between dynamic logical consequence and the semantics for *if* and *ought*.

3 Further Applications and Extensions

3.1 Order-Sensitivity

- A potential problem is the order-sensitivity of consistency; the premises of Forrester's paradox are inconsistent in case we reverse the order of premises, i.e. $m, m \Rightarrow \Box_d g, \Box_d \neg m \vDash_D \bot$
- One can arrive at a slightly modified conception of logical consequence that preserves the key ideas of the dynamic proposal and sidesteps the issue of order-sensitivity. Adopt the following semantics for conjunction:

$$[\![\phi \wedge \psi]\!]^{w,i} \text{ is true iff } [\![\phi]\!]^{w,i} \text{ is true and } [\![\psi]\!]^{w,i} \text{ is true}$$

  This notion of conjunction is internally static as opposed to internally dynamic (cf. Groenendijk and Stokhof (1991) for these notions, whose discussion is concerned with the issue of variable binding).

- Define validity for single-premise arguments and then generalize to cover multi-premise arguments:

  1. $\phi \vDash_{D*} \psi$ iff for all w and i such that $w \in i$: if $w \in [\![\phi]\!]^i$, then $w \in [\![\psi]\!]^{i+\phi}$
  2. $\phi_1, \ldots, \phi_n \vDash_{D*} \psi$ iff $\phi_1 \wedge \ldots \wedge \phi_n \vDash_{D*} \psi$

- This notion of logical consequence preserves the virtue of its predecessor—in particular its nonmonotonicity—but in addition guarantees that the premises of Forrester's paradox are dynamically consistent regardless of the order of premises.

3.2 Chisholm's Paradox

- The framework developed is in a position to offer an attractive solution to the paradox from Chisholm (1963). Consider:

  (5) Jones ought to go to the aid of his neighbors,

  (6) If Jones goes to the aid of his neighbors, then he ought to tell them he is coming,

  (7) If Jones does not go to the aid of his neighbors, then he ought not tell them he is coming,

  (8) Jones does not go to the aid of his neighbors.

- It seems that (5)-(8) are consistent and that none of these statements logically implies any other one. Chisholm observes that von Wright's (1951) classical deontic logic violates at least one these constraints regardless of how we formalize (5)-(8) but there is a more general reason for being interested in the case he describes. (5) and (6) are often taken to imply (via deontic detachment):

  (9) Jones ought to tell his neighbors that he is coming.

  But it also makes perfect sense to think that (7) and (8) imply (via factual detachment):

  (10) Jones ought not tell his neighbors that he is coming,

which contradicts (9). So it seems that we need to deny either deontic or factual detachment, which is too bad since both are very plausible. (See Loewer and Belzer (1983) and references therein for discussion; the labels "factual detachment" and "deontic detachment" go back to Greenspan (1975).)

- We can enforce deontic detachment via the following constraint on the deontic selection function:

  Weak Stability For all $i, i' \subseteq i$, if $d(i) \subseteq i'$, then $d(i) \subseteq d(i')$

  Weak Stability requires that strengthening with what is considered to be deontically ideal preserves deontically ideal worlds: doing what one ought to do at best absolves us from some our duties but does not create any new ones.

- We can endorse both factual and deontic detachment while avoiding Chisholm's paradox.
  - Deontic detachment licenses the inference of (9) from (5) and (6), but this inference is defeated by the premise (8) that triggers detachment of the contrary-to-duty obligation articulated by (7).
  - Precisely, assume that $\Sigma = \{\Box_d go, go \Rightarrow \Box_d tell\}$ and that $\Sigma' = \{\neg go, \neg go \Rightarrow \Box_d \neg tell\}$: then $\Sigma \vDash_D \Box_d tell$ and $\Sigma' \vDash_D \Box_d \neg tell$ as required by deontic and factual detachment, respectively. Yet there is no contradiction, for even though $\Sigma \vDash_D \Box_d tell$, we also have $\Sigma, \neg go \nvDash_D \Box_d tell$, which is compatible with deontic detachment since our deontic selection function is required to be realistic and thus $\Sigma, \neg go \nvDash_D \Box_d go$.
  - So under the assumption that Jones does not go, he ought not tell his neighbors that he is coming, but that is consistent with the intuition that he ought to tell his neighbors that he is coming in case he ought to go and also ought to tell them if he goes. What makes all this possible is that additional information in discourse and reasoning may defeat prior deontic truths and thus, in turn, inferences such as those licensed by deontic detachment.

- As I have demonstrated elsewhere (see Willer (forthcoming)), dynamic logical consequence also resolves Kolodny's and MacFarlane's recent miners paradox.

4 Appendix

4.1 Modus Ponens

- Notice first that some sentences are *locally invariant*:

  ϕ is locally invariant iff for all i: $[\![\phi]\!]^i = \varnothing$ or $[\![\phi]\!]^i = W$

  Whenever ϕ is locally invariant and $w \in [\![\phi]\!]^i$, then $i + \phi = i$ and thus $w \in [\![\phi]\!]^{i+\phi}$. Conditionals as well as deontic modals are locally invariant because their truth-value does not vary across possible worlds given some fixed informational parameter.

- To see why modus ponens is dynamically valid:
  - Modus ponens is dynamically valid just in case for all w and i such that $w \in i$: if $w \in [\![\phi \Rightarrow \psi]\!]^i$ and $w \in [\![\phi]\!]^{i+(\phi\Rightarrow\psi)}$, then $w \in [\![\psi]\!]^{(i+(\phi\Rightarrow\psi))+\phi}$.

- Due to the local invariance of conditionals, this condition is equivalent to the requirement that if $w \in [\![\phi \Rightarrow \psi]\!]^i$ and $w \in [\![\phi]\!]^i$, then $w \in [\![\psi]\!]^{i+\phi}$.
- Notice that ψ is to be evaluated with respect to the result of strengthening i with the conditional antecedent.
- But our semantics guarantees that $w \in [\![\psi]\!]^{i+\phi}$ whenever $w \in [\![\phi \Rightarrow \psi]\!]^i$. So the validity of modus ponens is an immediate result of the interaction between our semantic clauses and the dynamic conception of logical consequence.

4.2 Deontic Detachment

- Observation: given Weak Stability, $\Box_d go, go \Rightarrow \Box_d tell \vDash_D \Box_d tell$
- This holds just in case for all w and i such that $w \in i$: if $w \in [\![\Box_d go]\!]^i$ and $w \in [\![go \Rightarrow \Box_d tell]\!]^{i+\Box_d go}$, then $w \in [\![\Box_d tell]\!]^{(i+\Box_d go)+(go \Rightarrow \Box_d tell)}$. Since conditionals and deontic *ought* are locally invariant, this condition reduces to the requirement that whenever $w \in [\![\Box_d go]\!]^i$ and $w \in [\![go \Rightarrow \Box_d tell]\!]^i$, then $w \in [\![\Box_d tell]\!]^i$.
- Observe that $d(i) \subseteq i$ because of Realism and suppose that $w \in [\![\Box_d go]\!]^i$ and $w \in [\![go \Rightarrow \Box_d tell]\!]^i$: then $d(i) \subseteq [\![go]\!]^i$ and thus $d(i) \subseteq i + go$. Weak Stability then guarantees that $d(i) \subseteq d(i + go)$. But $d(i + go) \subseteq [\![tell]\!]^{i+go}$ and so $d(i) \subseteq [\![tell]\!]^{i+go}$. Since *tell* is f-persistent, $d(i) \subseteq [\![tell]\!]^i$ and thus $w \in [\![\Box_d tell]\!]^i$, as required.

4.3 The Miners Paradox

- Here is the miners paradox. Ten miners are trapped either in shaft A or in shaft B, but we do not know which one. Water threatens to flood the shafts. We only have enough sandbags to block one shaft but not both. If one shaft is blocked, all of the water will go into the other shaft, killing every miner inside. If we block neither shaft, both will be partially flooded, killing one miner.

Action	if miners in A	if miners in B
Block A	All saved	All drowned
Block B	All drowned	All saved
Block neither shaft	One drowned	One drowned

- Lacking any information about the miners' exact whereabouts, it seems right to say that

  (11) We ought to block neither shaft.

  However, we also accept that

  (12) If the miners are in shaft A, we ought to block shaft A,

  (13) If the miners are in shaft B, we ought to block shaft B.

  But we also know that

  (14) Either the miners are in shaft A or they are in shaft B.

  And (12)-(14) seem to entail

(15) Either we ought to block shaft A or we ought to block shaft B,

which contradicts (11)

- Kolodny and MacFarlane conclude that modus ponens is invalid but there is an alternative. It is easy to verify that disjunction elimination is dynamically invalid: $\phi \vee \psi, \phi \Rightarrow \chi, \psi \Rightarrow \psi \not\models_D \chi$. So (12)-(14) do not entail (15) even though conditional obligations detach.

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